Sample Paper – 2014

Subject – Mathematics
Class – XII

General Instructions

SECTION A:

1. If f: R $\to R$ be given by $f(x) = (3 - x^3)^{\frac{1}{3}}$ then find $fof(x)$

2. Simplify : $\cos\theta\begin{bmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{bmatrix} + \sin\theta\begin{bmatrix}\sin\theta & -\cos\theta \\ \cos\theta & \sin\theta\end{bmatrix}$

3. . Write the points where the function $f(x) = |x| + |x-1|$ is differentiable.

4. Find a matrix X such that B – 2A + X = O, where $A = \begin{bmatrix}5 & 3 \\ -3 & 1\end{bmatrix}$ and $B = \begin{bmatrix}0 & -2 \\ 3 & 1\end{bmatrix}$.

5. If $|\vec{a}| = 5$; $|\vec{b}| = 13$ and $|\vec{a} \times \vec{b}| = 25$, find $\vec{a}\,.\,\vec{b}$

6. Evaluate $\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$

7. Evaluate $\int \frac{(x^4 - x)^{\frac{1}{4}}\,dx}{x^5}$

8. Find $|\vec{a} - \vec{b}|$ *if two vectors* $\vec{a}$ *and* $\vec{b}$ *are such that* $|\vec{a}| = 2, |\vec{b}| = 3$ *and* $\vec{a}\,.\,\vec{b} = 4$

9. Find the value of $i.(j \times k) + j.(i \times k) + k.(i \times j)$

10. Find a unit vector perpendicular to each of the vectors $(\vec{a} + \vec{b})$ *and* $(\vec{a} - \vec{b})$ *where*

$\vec{a} = i + j + k$ *and* $\vec{b} = i + 2j + 3k$

SECTION B:

11. Let f: N $\rightarrow R$ be a function defined as $f(x) = 4x^2 + 12x + 15$ .Show that f: N $\rightarrow S$ where S is the range of f, is invertible .Find the inverse of f .

12. Solve for x : $\sin^{-1}(1-x) - 2\sin^{-1} x = \frac{\pi}{2}$. OR

Prove that $\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right] = \frac{\pi}{4} + \frac{1}{2}\cos^{-1} x^2, \frac{-1}{\sqrt{2}} \le x \le 1$

13. If x , y, z are different and $\Delta = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$ *then show that* $1+xyz = 0$

14. If the function $f(x) = \begin{cases} 1 : x \le 3 \\ ax + b ; 3 < x < 5 \\ 7 ; x \ge 5 \end{cases}$ is continuous at x = 3 and x = 5, then find the value of a & b.

15. *If* $x\sqrt{1+y} + y\sqrt{1+x} = 0$ *for* $-1 < x \le 1$ *provethat* $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$ OR

If $y = e^{a\cos^{-1} x}, -1 \le x \le 1$ *show that* $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2 y = 0$

16. Evaluate: $\int_{\frac{1}{2}}^{2} \frac{1}{x} \sin\left(x - \frac{1}{x}\right) dx$ and hence find : $\int_{\frac{1}{n}}^{n} \frac{1}{x} \sin\left(x - \frac{1}{x}\right) dx$, where n is positive integer.

17.Evaluate: $\int \frac{dx}{(\sin x - 2\cos x)(2\sin x + \cos x)}$ OR

Prove that $\int_0^{\frac{\pi}{4}} \log(1 + \tan\theta) d\theta = \frac{\pi}{8} \log 2$

18. Solve $(\tan^{-1} y - x)dy = (1+y^2)dx$ OR

Solve: $(1 + e^{x/y})\, dx + e^{x/y}(1 - x/y)\, dy = 0$

19. Find the intervals in which the function f(x) = sin x - cos x is increasing or decreasing on $[0, 2\pi]$. OR

Prove that the curves $y^2 = 4ax$ and $xy = c^2$ cut at right angles if $c^4 = 32a^4$

20. *Let* $\vec{a}, \vec{b}, \vec{c}$ *be three vectors* $|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 5$ *and each one of them is being*

Perpendicular to the sum of the other two find $|\vec{a} + \vec{b} + \vec{c}|$ OR

If with reference to the right handed system of mutually perpendicular unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$, $\vec{\alpha} = 3\hat{i} - \hat{j}$, $\vec{\beta} = 2\hat{i} + \hat{j} - 3\hat{k}$ then express $\vec{\beta}$ in the form of $\vec{\beta} = \vec{\beta_1} + \vec{\beta_2}$, where $\vec{\beta_1}$ is parallel to $\vec{\alpha}$ and $\vec{\beta_2}$ is perpendicular to $\vec{\alpha}$. (hots)

21. Find the shortest distance between the lines whose vector equations are

$\vec{r} = (1 - t)\, i + (t - 2)\, j + (3 - 2t)k$ *and* $\vec{r} = (s + 1)\, i + (2s - 1)\, j - (2s + 1)k$ OR

Find the image of the point P(6,5,9) on the plane determined by the points

A (3,-1,2) , B (5,2,4) and C (-1,-1,6)

22. The probability of a shooter hitting a target is 3/4. How many minimum number of times must he/ she fire so that the probability of hitting the target at least once is more than 0.99?

SECTION C:

23. Using elementary transformation find the inverse of the matrix $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

24. Prove that of all the triangles inscribed in a circle ,the equilateral triangle triangle has the maximum area. OR

A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost .Its semi vertical angle is tan-1 (0.5) .Water is poured in to it at a constant rate of 5 cubic meter per hour .Find the rate at which the level of the water is rising at an instant when the depth of the water tank is 4m.

25. Prove that $\int_0^{\frac{\pi}{2}} \log(\sin x)dx = \int_0^{\frac{\pi}{2}} \log(\cos x)dx = \frac{-\pi}{2}\log 2$ OR

Evaluate limit as a sum $\int_2^4 \frac{2}{3}x^2 - \frac{3x}{2} + 4dx$

26. Find the area of the smaller region bounded by the ellipse

$\frac{x^2}{16} + \frac{y^2}{9} = 1$ *and the line* $\frac{x}{4} + \frac{y}{3} = 1$

27. Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2,3 or 4 she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head, what is the probability that she threw 1, 2,3 or 4 with the die.

28. A line makes angles α, β, γ *and* δ with the diagonals of a cube , prove that

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$

29. A manufacturer of patent medicines is preparing a production plan on medicines A and B. There is sufficient raw material available to make 20,000 bottles of A and 40,000 bottles of B, but there are only 45,000 bottles into which either of medicines can be put. Further, it takes 3 hours to prepare enough material to fill 1,000 bottles of A and it takes one hour to prepare enough material to fill 1,000 bottles of B and there are 66 hours available for this operation. The profit is Rs.8 per bottle for A and Rs. 7 per Bottle for B. How should the manufacturer schedule his production in order to maximize his profit.