SAMPLE PAPER -2015
Class – XII
Subject – MATHEMATICS

Time : 3 hrs **Max. Marks : 100**

General Instructions:

(i) All questions are compulsory.

(ii) The question paper consists of 29 questions divided into three sections – A, B and C. Section A comprises of 10 questions of 1 mark each; Section B comprises of 12 questions of 4marks each and Section C comprises of 7 questions of 6 marks each.

SECTION – A

1. Show that the binary operation $*$ defined by $a*b = ab + 1$ on Q is commutative.

2. Solve : $\tan^{-1}2x + \tan^{-1}3x = \pi/4$.

3. Find a matrix X such that B – 2A + X = O, where $A = \begin{bmatrix} 5 & 3 \\ -3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -2 \\ 3 & 1 \end{bmatrix}$.

4. If A is a square matrix of order 3 such that $|\text{adj } A| = 64$, find $|A|$.

5. Construct a 2×2 matrix whose elements a_{ij} are given by: $a_{ij} = \frac{(2i + j)^2}{2}$.

6. Evaluate $\int \frac{dx}{x\cos^2(1+\log x)}$.

7. Evaluate $\int_{-\pi/2}^{\pi/2} \sin^5 x \cos^4 x\, dx$

8. If $|\vec{a}| = 5$; $|\vec{b}| = 13$ and $|\vec{a} \times \vec{b}| = 25$, find $\vec{a} \cdot \vec{b}$

9. The Cartesian equation of a line AB is $\frac{2x-1}{\sqrt{3}} = \frac{y+2}{2} = \frac{z-3}{3}$. Find the direction cosines of a line parallel to AB.

10. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$; $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$, find a unit vector parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$

SECTION-B

11.If $f : R-\left\{\frac{7}{5}\right\} \to R-\left\{\frac{3}{5}\right\}$ be defined as $f(x)=\frac{3x+4}{5x-7}$ and $g : R-\left\{\frac{3}{5}\right\} \to R-\left\{\frac{7}{5}\right\}$ bedefined as $g(x)=\frac{7x+4}{5x-3}$.Show that $g \circ f = I_A$ and $f \circ g = I_B$ where B= $R-\left\{\frac{3}{5}\right\}$ and A=R-$\left\{\frac{7}{5}\right\}$.

12.Using properties of determinants, prove that

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$$

13.Prove that : $2\tan^{-1}\frac{1}{5}+\sec^{-1}\frac{5\sqrt{2}}{7}+2\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$.

OR

Solve for x : $\sin^{-1}(1-x)-2\sin^{-1}x=\frac{\pi}{2}$.

14 For what value of k is the following function continuous at x = 2?

$$f(x)=\begin{cases} 2x+1; x<2 \\ k; x=2 \\ 3x-1; x>2 \end{cases}$$

15.If $x = 2\cos\theta - \cos 2\theta$ and $y = 2\sin\theta - \sin 2\theta$ find $\frac{d^2y}{dx^2}$ at $\theta=\frac{\pi}{2}$.

OR

If y = $[\log(x + \sqrt{1+x^2})]^2$, show that $(1+x^2)\frac{d^2y}{dx^2}+x\frac{dy}{dc}-2=0$.

16. Find the intervals in which the function f(x) = sin x – cos x; $0 \le x \le 2\pi$

(i) is increasing (ii) is decreasing.

17.Evaluate $\int \frac{x^3 \sin(\tan^{-1}x^4)}{1+x^8}dx$.**OR** Prove that $\int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\cos^4 x+\sin^4 x}dx=\frac{\pi^2}{16}$

18.Solve the following differential equation: $(1+x^2)\frac{dy}{dx}+y=\tan^{-1}x$.

19.Find the particular solution of the differential equation $\frac{dy}{dx}-\frac{y}{x}+\operatorname{cosec}\left(\frac{y}{x}\right)=0$, given that y(1) = 0.

20. If $\vec{a}$ and $\vec{b}$ are two vectors such that $\vec{a}.\vec{b}=\vec{a}.\vec{c}$, $\vec{a}\times\vec{b}=\vec{a}\times\vec{c}$ and $\vec{a}\neq 0$, then prove that $\vec{b}=\vec{c}$.

21.Find the shortest distance between the lines $\frac{x-1}{2}=\frac{y-1}{-1}=\frac{z}{1}$ and $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+1}{2}$.

22.A box contains 12 bulbs of which 3 are defective. If 3 bulbs are drawn from the box at random, find the probability distribution of X, the number of defective bulbs drawn. Hence compute the mean of X.

SECTION-C

23. Given that A = $\begin{vmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{vmatrix}$ and B = $\begin{vmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{vmatrix}$ find AB. Hence using this product solve the system of equations : $x - y + z = 4$, $x - 2y - 2z = 9$, $2x + y + 3z = 1$

OR

Using elementary row transformation, find the inverse of the matrix $\begin{bmatrix} 2 & -1 & 4 \\ 4 & 0 & 2 \\ 3 & -2 & 7 \end{bmatrix}$.

24. Show that a right circular cylinder, which is open at the top and has a given surface area, will have the greatest volume if its height is equal to the radius of its base.

OR

Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is 8/27 of the volume of the sphere.

25. Evaluate: $\int \sqrt{\tan x}\, dx$.

26. Using the method of integration, find the area of the region bounded by the lines $2x + y = 4$, $3x - 2y = 6$ and $x - 3y + 5 = 0$.

OR

Make a rough sketch of the region given below and find the area using the method of integration : $\{(x, y); 0 \le y \le x^2 + 3, 0 \le y \le 2x + 3, 0 \le x \le 3\}$

27. Find the image of the point (1, 2, 3) in the plane $x + 2y + 4z = 38$. Also find the perpendicular distance from the point to the plane.

OR

A line makes angles α, β, γ and δ with the diagonals of a cube, prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = 4/3$

28. An aeroplane can carry a maximum of 200 passengers. A profit of Rs. 400 is made on each first class ticket and a profit of Rs. 300 is made on each economy class ticket. The airline reserves at least 20 seats for first class. However, at least 4 times as many passengers prefer to travel by economy class to by the first class. Determine how many of each type ticket must be sold in order to maximize the profit for the airline. What is the maximum profit? Frame an L.P.P and solve it graphically.

29. In a bolt factory, machines A, B and C, manufacture respectively 25%,35%, 40% of the total bolts. Of their output 5%,4%&2% respectively are defective bolts. A bolt is drawn at random and is found to be defective. Find the probabilitythat it is manufactured by machine B.