

**Sample Paper – 2015**  
**Class – XII**  
**Subject – Mathematics**

**Time:3 Hours**

**Max Marks 100**

**General Instructions**

- 1 All questions are compulsory
- 2 Q 1 – 10 carries 1 marks, Q 11 – 22 carries 4 marks Q-23to 29 carries 6 marks

1. The function  $f: \mathbb{R} \rightarrow \mathbb{R}$  defined as  $f(x) = 10x+7$ . find the function  $g: \mathbb{R} \rightarrow \mathbb{R}$  such that  $g \circ f = f \circ g = I_{\mathbb{R}}$
2. Prove that in a skew symmetric matrix all the entries of a diagonal are .
3. If  $\tan^{-1}(x) = \sin^{-1}(1/2)$  find  $x$
4. Given  $A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$  verify  $2A^{-1} = I - A$
5. Solve the differential equation  $dy/dx = \sin(x+y)$
6. Using Rolle's theorem find the points on the curve  $y = x^2$ ,  $x \in [-2,2]$ , where the tangent is parallel to the  $x$  axis.
7. Evaluate  $\int \operatorname{cosec} x \, dx$
8. If "a" is a unit vector  $(x + a) \cdot (x - a) = 8$
9. Find the equation of a plane parallel to  $x$  axis and has intercepts 5 and 7 on  $y$  and  $z$  axis respectively.
10. If  $a = 5i-j-3k$ ,  $b = i +3j-5k$ , then show that the vectors  $a+b$  and  $a-b$  are orthogonal.
11. Show that  $f: \mathbb{R} \rightarrow A$  defined by  $f(x) = x^2/x^2 + 1$  is a surjection. Find  $A$ .
12. Solve for  $x$ .  $\tan^{-1}(x+1) + \tan^{-1}(x) + \tan^{-1}(x-1) = \tan^{-1}3$
13. If  $f$  is continuous at  $x = \pi/2$  Find  $a$  and  $b$   
 $f(x) = \frac{1 - \sin^3 x}{3\cos^2 x}$  if  $x < \pi/2$   
 $= a$  if  $x = \pi/2$   
 $= \frac{b(1 - \sin x)}{(\pi - 2x)^2}$  if  $x > \pi/2$
14. Show that  $\begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix} = 9b^2(a+b)$
15. Differentiate  $\tan^{-1} [\sqrt{1-\cos x}/1+\cos x]$  with respect to  $\tan^{-1}x$
16.  $\int \tan^{-1} \sqrt{x} \, dx$
17. Form the differential equation representing the family of curves  $y^2 - 2ay + x^2 = a^2$  (or) Prove that the differential equation is homogeneous and solve it  $2xy \, dx + (x^2 + 2y^2) \, dy = 0$
18. Evaluate using properties  $\int_1^1 \log \{2+x/2-x\} \, dx$
19. The radius of the balloon is increasing at the rate of 10cm/sec. At what rate is the surface area of the balloon is increasing when its radius is 15cm
20. Find the area of the parallelogram whose diagonals are  $d_1 = 3i+j-2k$   $d_2 = i-3j+4k$

21. Find the distance between the lines  $r = i+2j-4k+\lambda(2i+3j+6k)$  and  $r = 3i+3j-5k +\mu((2i+3j+6k)$
22. A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of no of tails. Find mean and variance
23. If  $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$  prove that  $A^2 - 4A - 5I = 0$  Hence find  $A^{-1}$
24. Find the equation of the line through the point (3,4) which cuts from the first quadrant of triangle of minimum area (or)  
A wire of length 36 cm is cut into two pieces. One of the pieces is turned in the form of a square and the other in the form of an equilateral triangle. Find the length of the each piece so that the sum of the areas of the two be minimum.
25. Find the area enclosed by the region in the first quadrant enclosed by the ellipse  $x^2/4 + y^2/36 = 1$  and the line  $3x+y = 6$ (or)  
Find the area bounded by the curve  $y^2 = 4a(x-1)$  and the lines  $x = 1$  and  $y = 4a$ .
26. Find the limit of sum using integration of  $\int_0^3 x^2 - 2x+2 dx$
27. Find the equation of the plane passing through (1,2,-4) and parallel to the lines  $r = i+2j-4k+\lambda(2i+3j+6k)$ ,  $r = i-3j+5k+\mu(i+j-k)$  and also find the perpendicular distance of this plane from the origin, and find the direction cosines of its normal.
28. A dealer wishes to purchase a number of fans and sewing machines. He has only Rs 5760 to invest and has space for at the most 20 items. A fan costs him Rs 360 and sewing machine Rs 240. He expects to sell a fan at a profit of Rs 22 and a sewing machine for a profit of Rs 18. Assuming that he can sell all the items he buys, how should he invest his money to maximize his profit. Solve it graphically.
29. Suppose a girl throws a die. If she gets 5 or 6 she tosses the coin 3 times and notes the number of heads. If she gets 1,2,3 or 4 she tosses a coin once and notes whether a head or a tail is obtained if she obtained exactly one head what is the probability that she threw 1,2,3 or 4 with a die.