

SAMPLE PAPER-2015
Class – XII
Subject – MATHEMATICS

Time: 3 hrs

M.M: 100

General Instructions :

1. All question are compulsory.
2. The question paper consists of 29 questions divided into three sections A,B and C. Section – Acomprises of 10 question of 1 mark each. Section – B comprises of 12 questions of 4 marks each andSection – C comprises of 7 questions of 6 marks each .
3. There is no overall choice. However, internal choice has been provided in 4 question of four marks and2 questions of six marks each. You have to attempt only one lf the alternatives in all such questions.

SECTION-A

1. Find the value of $\tan^{-1} \left[2\cos \left(2 \sin^{-1} \frac{1}{2} \right) \right]$.
2. If $\int_0^1 (3x^2 + 2x + k)dx=0$. Find the value of k.
3. If $A = \begin{bmatrix} 0 & i \\ i & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Find the value of k.
4. If the binary operation*on the set of integers Z is defined by $a*b=a+3b^2$. Find the value of $3*4$.
5. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then find the angle between \vec{a} and \vec{b} .
6. Find λ ,if the lines $\frac{1-x}{3} = \frac{7y-18}{2\lambda} = \frac{5z-10}{11}$ and $\frac{7-7x}{3\lambda} = \frac{y-5}{1} = \frac{6-z}{5}$ are perpendicular.
7. Evaluate $\int \frac{dx}{x \cos^2(1+\log x)}$.
8. If $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix}$, Find the value of k.
9. If A is a Non-singular matrix of order 3 and $|\text{adj}A| = |A|^k$, Find the value of k.
10. Find the angle between two vectors \vec{a} and \vec{b} having the same magnitude $\sqrt{2}$ and their scalar product is -1.

SECTION-B

11. Find the image of the point having position vector $(i+3j+4k)$ in the plane $\vec{r} \cdot (2i-j+k) + 3 = 0$.
12. Evaluate $\int \frac{x^2+x+1}{(x+2)(x^2+1)} dx$ OR $\int_0^{\frac{\pi}{2}} \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx$.
13. The scalar product of the vector $(i+j+k)$ with the unit vector along the sum of vector $(2i+4j-5k)$ and $(\lambda i+2j+3k)$ is equal to one. Find the value of λ .
14. Is $f(x) = |x-1| + |x-2|$ continuous and differentiable?
OR

If $(\cos x)^y = (\sin y)^x$, find $\frac{dy}{dx}$

15. Form the differential equation of the family of circles touching the x-axis at origin.

16. Using the properties of determinant, prove that

$$\begin{vmatrix} a^2 + 1 & ab & ac \\ ab & b^2 + 1 & bc \\ ca & cb & c^2 + 1 \end{vmatrix} = (1+a^2 + b^2 + c^2)$$

17. Find the particular solution satisfying the given condition for the differential equation

$$\frac{dy}{dx} - \frac{y}{x} + \operatorname{cosec} \frac{y}{x} = 0 \text{ given } y=0 \text{ when } x=1$$

18. Let R_+ be the set of all non-negative real numbers. Let $f: R_+ \rightarrow [4, \infty)$; $f(x)=x^2+4$. Show that f is invertible and find f^{-1} .

19. Find the value of x for which $f(x)=[x(x-2)]^2$ is an increasing function. Also find the point on the curve at which tangents are parallel to x -axis.

20. A football match may be either won, drawn or lost by the host country's team. So there are three ways of forecasting the result of any match, one correct and two incorrect. Find probability forecasting at least three correct result for four matches.

21. If $x=a(\cos\theta + \log \tan \frac{\theta}{2})$ and $y=a \sin\theta$, find the value of $\frac{d^2y}{dx^2}$ at $\theta=\pi/4$

OR

$$\text{If } y = \cos^{-1} \left[\frac{2x-3\sqrt{1-x^2}}{\sqrt{13}} \right], \text{ find } \frac{dy}{dx}.$$

22. Prove that $\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3) = \pi$

OR

$$\text{Prove that } \tan^{-1} \left(\frac{3}{4} \right) + \tan^{-1} \left(\frac{3}{5} \right) - \tan^{-1} \left(\frac{8}{19} \right) = \frac{\pi}{4}$$

SECTION-C

23. Find the inverse of the matrix $\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ by using elementary row transformation.

24. A small firm manufactures gold rings and chains. The total number of rings and chains manufactured per day is at most 24. It takes 1 hour to make a ring and 30 minutes to make a chain. The maximum number of hours available per day is 16. If the profit on a ring is Rs.300 and that on a chain is Rs.190, find the number of rings and chains that should be manufactured per day, so as to earn the maximum profit. Make it as L.P.P. and solve it graphically.

25. Using integration, find the area of the region bounded by the curve $x^2=4y$ and the line $x=4y-2$.

OR

Using integration, find the area of the region bounded by the curve $y^2=4a^2(x-1)$ and $x=1$ and $y=4a$.

26. Evaluate $\int_1^3 (x^2+x)dx$ as the limit of sums.

27. A wire of length 28 m is to be cut into two pieces. One of the two pieces is to be made into a square and the other into a circle. What should be the lengths of two pieces so that the combined area of circle and square is minimum?

28. Find the equation of the plane passing through the point $P(1,1,1)$ and containing the line $\vec{r} = (-3\hat{i} + \hat{j} + 5\hat{k}) + \lambda(3\hat{i} - \hat{j} - 5\hat{k})$. Also, show that the plane contains the line $\vec{r} = (-\hat{i} + 2\hat{j} + 5\hat{k}) + \mu(\hat{i} - 2\hat{j} - 5\hat{k})$.

29. A man is known to speak the truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.