Sample Paper – 2015

Class –XII
Subject –Mathematics

General instructions:

(i) All questions are compulsory.
(ii) Question number 1 to 10 carry 1 mark each.
(iii) Questions numbers 11 to 22 carry four marks each.
(iv) Question number 23 to 29 carry six marks each.
(v) There is no over-all choice. However, internal choice has been provided in some questions. You have to answer only one of them.
(vi) Use of calculator is not permitted.

SECTION--A

1 Let f: R $\rightarrow$R such that f(x) = x + 7 and g: R$\rightarrow$R Such that g(x) = x - 7 .Find gof .

2 Write the identity element for the binary operation * defined on the set R of real numbers by the rule a * b = $\frac{3ab}{8}$, for all a, b $\in$ R.

3 If $\begin{bmatrix} x+3y & y \\ 7-x & 4 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ 0 & 4 \end{bmatrix}$, Find the value of x and y.

4 Evaluate: $\begin{bmatrix} a+ib & c+id \\ -c+id & a-ib \end{bmatrix}$

5 If A is a square matrix of order 3 such that $|adjA| = 64$, Find $|A|$.?

6 What is the principal value of $\sin^{-1}\left(\sin\frac{5\pi}{6}\right) + \cos^{-1}\left(\cos\frac{\pi}{6}\right)$?

7 Solve for x : $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \frac{1}{2}\tan^{-1}x, x > 0$

8 Write the value of $\int e^{3\log x}.(x^4)\,dx$.

9 Find the slope of the curve $y = x^3 - x$ at x = 2

10 Find $\frac{d^2y}{dx^2}$, if $y = x^3 + \tan x$

SECTION--B

11 Let T be the set of all triangle in a plane with R a relation in T given by R = {(T_1,T_2):T_1 is similar to T_2}.Show that R is an equivalence relation.

12 Write the value of $2\tan^{-1}\left(\frac{1}{5}\right) + \sec^{-1}\left(\frac{5\sqrt{2}}{7}\right) + 2\tan^{-1}\left(\frac{1}{8}\right)$ **or**

Solve for x; $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}; |x| < 1$

13.Using properties of determinants, prove that $\begin{vmatrix} (y+z)^2 & x^2 & x^2 \\ y^2 & (z+x)^2 & y^2 \\ z^2 & z^2 & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^3$. ?**or**

Prove that $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & cb & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$?

14.Determine the value of the constant k, so that the function $f(x)=\begin{cases}\dfrac{k\cos x}{\pi-2x} & ; x\neq\dfrac{\pi}{2}\\ 3 & ; x=\dfrac{\pi}{2}\end{cases}$ is continuous.

15. If $\sqrt{1-x^6}+\sqrt{1-y^6}=a\left(x^3-y^3\right)$, prove that $\dfrac{dy}{dx}=\dfrac{x^2}{y^2}\sqrt{\dfrac{1-y^6}{1-x^6}}$

orProve that $\dfrac{d}{dx}\left[\dfrac{x}{2}\sqrt{a^2-x^2}+\dfrac{a^2}{2}\sin^{-1}\dfrac{x}{a}\right]=\sqrt{a^2-x^2}$

16. If $x=a\cos\theta, y=b\sin\theta$, show that $\dfrac{d^2y}{dx^2}=-\dfrac{b^4}{a^2y^3}$

17.If $x=e^{\frac{y}{x}}\left(a+bx\right)$, prove that $x^3\dfrac{d^2y}{dx^2}=\left(x\dfrac{dy}{dx}-y\right)^2$

18. Water is dripping out from a conical funnel of semi-vertical angle $\dfrac{\pi}{4}$ at the uniform rate of 2 $cm^2/\sec$ in its surface area through a tiny hole at the vertex in the bottom. When the slant height of the water is 4 cm, find the rate of decrease of the slant height of the water.

19. Given that for the function f defined by $f(x)=x^3+bx^2+ax, x\in[1,3]$, Rolle's theorem holds with $c=2+\dfrac{1}{\sqrt{3}}$. Find the value of a and b.

20. Find the equations of the tangent and the normal to the curve $x=1-\cos\theta,\ y=\theta-\sin\theta$ at $\theta=\dfrac{\pi}{4}$.

21.Show that $f\left(x\right)=\tan^{-1}\left(\sin x+\cos x\right)$ is an increasing function on the interval $\left(0,\pi/4\right)$

22.Evaluate $\int\left[\log(\log x)+\dfrac{1}{(\log x)^2}\right]dx$ **or** $\int\sqrt{\tan\theta}\,d\theta$

SECTION--C

23. $A=\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}$, find A^{-1} and hence solve the system of equations $\begin{aligned}x+2y+z&=4\\ -x+y+z&=0\\ x-3y+z&=2\end{aligned}$ **or**

Find the inverse of the matrix $A=\begin{bmatrix}1 & 2 & -2\\ -1 & 3 & 0\\ 0 & -2 & 1\end{bmatrix}$ by using elementary row transformations

24.A window is in the form of a rectangle above which there is a semi-circle. If the perimeter of the window is p cm, show that the window will allow the maximum possible light only when the radius of the semicircle is $\dfrac{p}{\pi+4}$ cm

25. Show that the volume of the greatest cylinder which can be inscribed in a cone of height h and semi vertical angle α is $\dfrac{4}{27}\pi h^3\tan^2\alpha$ **or**

An open box with a square base is to be made out of a given quantity of card board of area c^2 square units . Show that the maximum volume o the box is $\frac{c^3}{6\sqrt{3}}$

26. Evaluate $\int_0^{\pi} \log(1+\cos x)dx$

27. Evaluate $\int_1^4 (x^2 - x)dx$ as limit of sum.

28.Find the area of the region $\left[(x,y): y^2 \le 4x, 4x^2 + 4y^2 \le 9\right]$

29.Using the method of integration, find the area of the region bounded by the lines
2x + y = 4, 3x – 2y = 6 and x – 3y + 5 = 0.