Sample Paper – 2015
Class – XII
Subject – Mathematics

TIME : 3 hours **Max Marks :- 100**

General Instructions :

(i) All questions are compulsory.

(ii) The question paper consists of 29 questions divided into **3** sections **A, B** and **C**. Section A comprises of 10 questions of 1 mark each, Section B comprises of 12 questions of 4 marks each , Section C comprises of 7 questions of **6** marks each.

(iii) All questions in section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(iv) There is no overall choice. However internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

Section–A

Question numbers 1 to 10 carry 1 mark each.

1. If $f(x)$ is an invertible function, find the inverse of $f(x) = \dfrac{3x-2}{5}$.

2. What is the principal value of $\tan^{-1}\left(\tan\dfrac{3\pi}{4}\right)$.

3. If A = $\begin{bmatrix} 3 & 4 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$, find $B'A'$.

4. If $*$ be a binary operation defined on Q, given by $a * b = a + ab,\ \forall\, a, b \in$ Q. Is * commutative?

5. Evaluate $\displaystyle\int \frac{x^2}{1+x^3}\,dx$.

6. Give example of a function which is continuous at $x = 1$ but not differentiable at $x = 1$.

7. Evaluate : $\sin\left[\dfrac{\pi}{3} - \sin^{-1}\left(-\dfrac{1}{2}\right)\right]$.

8. If A is a invertible matrix of order 3 and $|A| = 6$, then find $|\text{adj}\,(A)|$.

9. Find the slope of tangent to the curve $y = x^2 - 2$ at the point whose abscissa is 2.

10. If $2\sqrt{x} + 3\sqrt{y} = a$ then find $\dfrac{dy}{dx}$ at $\left(\dfrac{1}{4}, \dfrac{1}{4}\right)$.

Section–B

Question numbers 11 to 22 carry 4 marks each.

11. Let Z be the set of integers and R be the relation defined on Z by R = { $(a, b) : a, b \in Z$, $|a - b|$ is divisible by 5}. Prove that R is an equivalence relation.

Or

Consider the binary operation $* : R \times R \to R$ and $o : R \times R \to R$ defined as $a * b = |a - b|$ and $a \text{ o } b = a$. Show that $a * (b \text{ o } c) = (a * b) \text{ o } (a * c) \quad \forall$ a, b, c $\in$ R

12. Using properties of determinants, prove the following

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2 .$$

13. Using differentials, find the approximate value of $\sqrt{0.037}$.

14. Solve for x : $\cos^{-1}\left(\frac{x^2-1}{x^2+1}\right) + \tan^{-1}\left(\frac{2x}{x^2-1}\right) = \frac{2\pi}{3}$.

Or

Prove that $\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right) = \frac{2b}{a}$.

15. Evaluate : $\int \frac{1}{\sqrt{5-4x-2x^2}}\, dx$.

16. Let $f : N \cup \{0\} \to N \cup \{0\}$ be defined by $f(n) = \begin{cases} n-1, & if\ n\ is\ odd \\ n+1, & if\ n\ is\ even \end{cases}$. Show that $f(x)$ is bijective. Also show that $f^{-1} = f$.

17. Express A = $\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$ as the sum of a symmetric matrix and a skew symmetric matrix.

Or

Let A = $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$. Using principle of mathematical induction, prove that $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ for every positive integer n.

18. Solve the Linear Programming Problem graphically.

Maximise and Minimise $z = 3x + 9y$ subject to the constraints

$x + 3y \le 60$; $x \le y$; $x + y \ge 10$; $x \ge 0$; $y \ge 0$.

19. Evaluate : $\int \frac{\sin 2x}{a\cos^2 x + b\sin^2 x} dx$

20. Determine the constants a and b, such that the function by $f(x) = \begin{cases} ax^2 + b, & x > 2 \\ 2, & x = 2 \\ 2ax - b, & x < 2 \end{cases}$ is continuous.

Or

Verify Rolle's Theorem for the function $f(x) = \sin 2x - 2 \sin x$, $x \in [0, \pi]$.

21. Prove that $\cos[\tan^{-1}\{\sin (\cot^{-1}x)\}] = \sqrt{\frac{x^2+1}{x^2+2}}$

22. A man 180 cm tall walks at a rate of 2 cm/sec away from the source of light that is 9 m above the ground. How fast is the length of his shadow increasing when he is 3 m away from the base of light?

Section–C

Question numbers 23 to 29 carry 6 marks each.

23. Find the inverse of the following matrix using elementary operations. A = $\begin{bmatrix} 2 & -1 & 4 \\ 4 & 0 & 2 \\ 3 & -2 & 7 \end{bmatrix}$

24. An aeroplane can carry a maximum of 200 passengers. A profit of Rs 400 is made on each first class ticket and a profit of Rs 300 is made on each second class ticket. The airline reserves at least 20 seats for first class. However, at least 4 times as many passengers prefer to travel by second class than by the first class. Determine how many tickets of each type must be sold in order to maximize the profit for the airline. Form an L.P.P. and solve it graphically.

Or

A dietician has to develop a special diet using two foods P and Q. Each packet (containing 30 g) of food P contains 12 units of calcium, 4 units of iron, 6 units of cholesterol and 6 units of vitamin A. Each packet of the same quantity of food Q contains 3 units of calcium, 20 units of iron, 4 units of cholesterol and 3 units of vitamin A. The diet requires at least 240 units of calcium, at least 460 units of iron and at most 300 units of cholesterol. How many packets of each food should be used to maximise the amount of vitamin A in the diet? What is the minimum amount of vitamin A?

25. Find the area of the greatest isosceles triangle that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with its vertex at one end of the major axis.

26. Find all points of discontinuity of the function $f(x) = \begin{cases} |x| + 3 & , \ \text{if} \quad x \leq 3 \\ -2x & , \ \text{if} \quad -3 < x \leq 3 \\ 6x + 2 & , \ \text{if} \quad x > 3 \end{cases}$.

27. If $x = a(\theta - \sin\theta)$ and $y = a(1 + \cos\theta)$, then find $\dfrac{d^2y}{dx^2}$ at $\theta = \dfrac{\pi}{3}$.

28. Find the intervals in which the function $f(x) = (x - 1)(x - 2)^2$ is (*a*) strictly increasing (*b*) strictly decreasing.

Or

Find the equation of tangent and normal to the curve given by $x = a\cos^3 t$ and $y = b\sin^3 t$ at a point where $t = \dfrac{\pi}{4}$.

29. Given two matrices A = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$. Verify that AB = 6I. Use this result to solve the system of linear equations $x - y = 3$; $2x + 3y + 4z = 17$; $y + 2z = 7$