SAMPLE PAPER -2015
SUBJECT:MATHEMATICS
CLASS:XII

Time Allowed: 180 Minutes **Max. Marks:100**

General instructions:

a) Note that all the questions are compulsory.
b) The question paper consists of 29 questions divided into three sections A, B and C. Section A comprises
of 10 questions of one mark each, Section B comprises of 12 questions of four marks each, and Section
C comprises of 7 questions of six marks each. All questions in section A are to be answered in one word,
one sentence or as per the exact requirements of the question.
c) There is no overall choice. However internal choice has been provided in some of the cases

<u>Section – A</u>

(Questions number 1 to 10 carry 1 marks each)

1. Find the principal value of. $\tan^{-1}\left[2cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$.
2. Solve $\tan^{-1}(1+x)+\tan^{-1}(1-x)=\frac{\pi}{2}(-1<x<1)$.
3. Find the order and degree of the differential equation.. $\sqrt[5]{log_{\ e}\left(1+\frac{d^2y}{dx^2}\right)}=x$.
4. Without actual expanding write the value of. $\begin{vmatrix} 0 & -54 & 98 \\ 54 & 0 & 67 \\ -98 & -67 & 0 \end{vmatrix}$
5. If A'=$\begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and B=$\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$,then find (A+2B).
6. Differentiate w.r.t. 'x', cos $(2^{log_2 x}$.
7. Find the distance between the parallel planes$\underset{r}{\rightarrow}=\left(2i^{-j^{+3k^{x}}}\right)=4\ and\ 6x-3y+9z+13=0$.
8. A=$\begin{bmatrix} 1 & 0 & 1 \\ k & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix}$, then write$|3\ A|$ and find the value of k, if A is singular matrix.
9. Find a vector in the direction of given vector i^+j^+k^having magnitude 8 units.
10. Find the sine of the angle between the vectors$\underset{a}{\leftarrow}=2i^\wedge-j^\wedge+3k^\wedge$ and$\underset{b}{\rightarrow}=i^\wedge+3j^\wedge+2k^\wedge$..

<u>Section-B</u>

(*Questions number 11 to 22 carry 4 marks each.***)**

11. Let F:R→and f(x)=$\begin{cases} 1 & x>0 \\ 0 & x=0 \\ -1 & x<0 \end{cases}$ $and\ g(x)=[x], then\ does\ fog\ and\ gof\ coinside\ in$ $[-1,0]$?

OR

Given X$\neq \emptyset, let\ *$: P(X) x P(X)$\rightarrow P(X)$be defined as A*B=(A-B)∪(B-A),$\forall A$,B∈P(X) show that $\phi$is the identity element for the operation * and all the elements A of P(X) are invertible withA^{-1}=A.

12. By using the properties of determinants prove that :

$$\begin{vmatrix} \mathbf{1+a} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{1+b} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} & \mathbf{1+c} \end{vmatrix} = \boldsymbol{abc}\left(\frac{\mathbf{1}}{\boldsymbol{a}} + \frac{\mathbf{1}}{\boldsymbol{b}} + \frac{\mathbf{1}}{\boldsymbol{c}} + \mathbf{1}\right).$$

(2)

13. Show that $\tan^{-1} x \left[\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}\right] = \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x. where -\frac{1}{\sqrt{2}} \leq x \leq 1.$

OR

Show that,$\sin^{-1}\left[\frac{12}{13}\right] + \cos^{-1}\left[\frac{4}{5}\right] + \tan^{-1}\left[\frac{63}{16}\right] = \pi$

14. If**cos** y= x cos(a+y), prove that $\frac{\boldsymbol{dy}}{\boldsymbol{dx}} = \frac{\boldsymbol{\cos^2(a+y)}}{\mathbf{\cos a}}$

15. Verify the applicability of the Rolle's Theorem for the function f(x)=$\boldsymbol{\sin^4 x + \cos^4 x\ on} \left[\mathbf{0}, \frac{\boldsymbol{\pi}}{\mathbf{2}}\right) /$

16. For what value of a and b, the function defined as:

$$F(x)=\begin{cases} \boldsymbol{3ax+b; if\ x} > 1 \\ \mathbf{11}; \boldsymbol{if\ x = 1} \\ \boldsymbol{5ax-2b; if\ x} < 1 \end{cases} \quad \boldsymbol{is\ continuous\ at\ x = 1}.$$

17.Evaluate:$\int \frac{\sin^8 \boldsymbol{x} - \cos^8 \boldsymbol{x}}{\mathbf{1-2}\sin^2 \boldsymbol{x}.\cos^2 \boldsymbol{x}} \boldsymbol{dx}.$

18. Evaluate:$\int \frac{5x+3}{\sqrt{\boldsymbol{x^2}+4x+10}} dx$. What should be the ideal role of youth in national integration?

19. Evaluate:$\int_0^{\frac{\pi}{2}} \log(\tan x + \cot x) dx.$

20. Leta=i^+4j^+2k^, b=3i^-2j^+7k^ and c=2i^-j^+4k^, find a vector d which is perpendicular to both a and b, and c.d.=15. "Directionless youth is the burden on nation" comment on it.

OR

A girl walks 5 km towards west, and then she walks 3 km in a direction 60° east of north and stops. Determine the girl's displacement from her initial point of departure.*Respect the girl implies respect the nation comment on it.*

21. Find the length of the perpendicular drawn from the point (2, -3,1) to the line $\frac{x+1}{2} = \frac{3-y}{-3} = \frac{-2z-4}{2}$..

22. Consider the experiment of tossing a coin. If the coin shows head, toss it again but if it shows tail, then throw a die. Find the conditional probability of the event that 'the die shows a number greater than 4' given that 'there is at least one tail'.

OR

Let E and F be two independent events. The probability that exactly one of them occurs is $\frac{11}{25}$and the probability of none of them occurring is$\frac{2}{25}$. If P(T) denotes the probability of the occurrence of the event T, then find the value of P(E) and P(F).

(3)

Section-C

(*Question number 23 to 29 carry 6marks each****)***

23. Using elementary transformations, find the inverse of $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$.

24. The total cost function and demand function of an item are given by C(x)=$\frac{x^3}{3} - 7x^2 + 111x + 5 - and\ p = 100 - x\ respectively$.Write the total revenue function. Find the number of items when the profit will be maximum. Find the maximum profit also.

OR

Find the equation of tangents to the curve y= cos (x + y), -2$\pi \leq x \leq 2\pi$ that are parallel to the line$x$ + 2y = 0.

25.Find the area lying above x-axis and included between the circle $x^2 + y^2 = 8x$ and inside the parabola$y^2 = 4x$.

Solve the differential equation$\left[\frac{e^{-2\sqrt{y}}}{\sqrt{y}} - \frac{x}{\sqrt{y}}\right]\frac{dy}{dx} = 1; (y \neq 0)$and y(1)=2.

OR

Solve the differential equation (x. dy- y .dx) y. sin$\left(\frac{y}{x}\right) = (y.dx + x.dy).x.cos\left(\frac{y}{x}\right)$.

26. Assume that the chance of a patient having a heart attack is 40%. It is alsoassumed that a meditation and yoga course reduce the risk of heart attack by30% and prescription of certain drug reduces its chances by 25%. At a time apatient can choose any one of the two options with equal probabilities. It is giventhat after going through one of the two options the patient selected at randomsuffers a heart attack. Find the probability that the patient followed a course ofmeditation and yoga?*Why Meditation and Yoga is necessary and sufficient thing for peace in mind and for good health.*

27. Find the vector equation of the line passing through (1, 2, 3) and parallel to the plane r =.i^-j^+2k^)=5 and 3x+y+z=6.

28. An aero plane can carry a maximum of 200 passengers. A profit of ₹1000 ismade on each executive class ticket and a profit of ₹600 is made on eacheconomy class ticket. The airline reserves at least 20 seats for executive class.However, at least 4 times as many passengers prefer to travel by economy classthan by the executive class. Determine how many tickets of each type must besold in order to maximize the profit for the airline. What is the maximum profit?*How one should respect the hard earn money of parents/guardians in a best economical way.*

Pre-board Examination 2012-13

Marking Scheme

S. No.	View Points	Marks
1.	$\frac{\pi}{4}$	**1**
2.	0	**1**
3.	Order: 2, Degree:1	**1**
4.	0	**1**
5.	$\begin{bmatrix} -4 & 5 \\ 1 & 6 \end{bmatrix}$	**1**
6.	- sin x	**1**
7.	$\frac{25}{3\sqrt{24}} units$	**1**
8.	**0**(zero)	**1**
9.	$\frac{8}{\sqrt{3}}\hat{i}+\frac{8}{\sqrt{3}}\hat{j}+\frac{8}{\sqrt{3}}\hat{k}$	**1**
10.	$\frac{3\sqrt{19}}{14}$	**1**
11.	fog(x)=f(g(x))=f(-1)=-1 again gof(x)=g(f(x))=g(-1)=[-1]=-1 thus for all x in[-1,0) fog(x)=gof(x) **OR** It is given that *: P(X) × P(X) → P(X) is defined as $A * B = (A - B) \cup (B - A)$ for every $A, B \in$ P(X). Let $A \in$ P(X). Then, we have: $A * \Phi = (A - \Phi) \cup (\Phi - A) = A \cup \Phi = A$ $\Phi * A = (\Phi - A) \cup (A - \Phi) = \Phi \cup A = A$ $\therefore A * \Phi = A = \Phi * A.$ $\quad A \in$ P(X) Thus, Φ is the identity element for the given operation*. Now, an element $A \in$ P(X) will be invertible if there exists $B \in$ P(X) such that $A * B = \Phi = B * A$. (As Φ is the identity element)	1+1/2 1+1/2 1 **1** **1**

	Now, we observed that. A*A=(A-A)∪ $(A-A)=\emptyset\cup\emptyset=\emptyset\forall A°\in P(X)$ Hence, all the elements A of P(X) are invertible with $A^{-1}=A$.	**1** **1**
12.	Multiply *abc* in rows and take *abc* common from column Now $R_1=R_1+R_2+R_3$ Then $R_3=R_3-R_1$ and $R_2=R_2-R_1$ then expand to get the result.	1 1 1 1
13.	$put\ x=\cos 2\theta \Rightarrow \theta=\frac{1}{2}\cos^{-1}x$ $=\tan^{-1}\left[\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right]$ $=\tan^{-1}\left[\frac{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}\right]$ $=\tan^{-1}\left[\frac{\lvert\cos\theta\rvert-\lvert\sin\theta\rvert}{\lvert\cos\theta\rvert+\lvert\sin\theta\rvert}\right]$ $=\tan^{-1}\left[\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\right]\quad \because -\frac{1}{\sqrt{2}}\le x\le 1$ $=\tan^{-1}\left[\frac{1+\tan\theta}{1-\tan\theta}\right]$ $=\tan^{-1}\left[\tan\left(\frac{\pi}{4}-\theta\right)\right]=\left(\frac{\pi}{4}-\theta\right)=\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x\right)$ *Hence Proved*	½ **1** **1** ½ **1**
14.	It is given that, $\cos y=x\cos(a+y)$ $\therefore \frac{d}{dx}[\cos y]=\frac{d}{dx}[x\cos(a+y)]$ $\Rightarrow -\sin y\frac{dy}{dx}=\cos(a+y)\cdot\frac{d}{dx}(x)+x\cdot\frac{d}{dx}[\cos(a+y)]$ $\Rightarrow -\sin y\frac{dy}{dx}=\cos(a+y)+x\cdot[-\sin(a+y)]\frac{dy}{dx}$ $\Rightarrow [x\sin(a+y)-\sin y]\frac{dy}{dx}=\cos(a+y) \quad ...(1)$ Since $\cos y=x\cos(a+y),\ x=\frac{\cos y}{\cos(a+y)}$ Then, equation (1) reduces to	**1** **1** **1**

	$\left[\frac{\cos y}{\cos(a+y)}\cdot\sin(a+y)-\sin y\right]\frac{dy}{dx}=\cos(a+y)$ $\Rightarrow\left[\cos y\cdot\sin(a+y)-\sin y\cdot\cos(a+y)\right]\cdot\frac{dy}{dx}=\cos^2(a+y)$ $\Rightarrow\sin(a+y-y)\frac{dy}{dx}=\cos^2(a+b)$ $\Rightarrow\frac{dy}{dx}=\frac{\cos^2(a+b)}{\sin a}$	**1**
15.	Continuous on $[0,\pi/2]$ Differentiable at $(0,\pi/2)$ $f(0)=f(\pi/2)=1$ $f'(c)=0$ will give c=π/4 ∈ (0,π/4)	**1** **1** **1** **1**
16.	L.H.L.=5a-2b and R.H.L.=3a+b a= 3 b=2	**1** **1** **1** **1**
17.	$\frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}=\frac{(\sin^4 x+\cos^4 x)(\sin^4 x-\cos^4 x)}{\sin^2 x+\cos^2 x-\sin^2 x\cos^2 x-\sin^2 x\cos^2 x}$ $=\frac{(\sin^4 x+\cos^4 x)(\sin^2 x+\cos^2 x)(\sin^2 x-\cos^2 x)}{(\sin^2 x-\sin^2 x\cos^2 x)+(\cos^2 x-\sin^2 x\cos^2 x)}$ $=\frac{(\sin^4 x+\cos^4 x)(\sin^2 x-\cos^2 x)}{\sin^2 x(1-\cos^2 x)+\cos^2 x(1-\sin^2 x)}$ $=\frac{-(\sin^4 x+\cos^4 x)(\cos^2 x-\sin^2 x)}{(\sin^4 x+\cos^4 x)}$ $=-\cos 2x$ $\therefore\int\frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}dx=\int-\cos 2x\,dx=-\frac{\sin 2x}{2}+C$	**1** **1** **1** **1**
18.	Let $5x+3=A\frac{d}{dx}(x^2+4x+10)+B$ $\Rightarrow 5x+3=A(2x+4)+B$ Equating the coefficients of *x* and constant term, we obtain	**1**

$$2A = 5 \Rightarrow A = \frac{5}{2}$$

$$4A + B = 3 \Rightarrow B = -7$$

$$\therefore 5x + 3 = \frac{5}{2}(2x+4) - 7$$

1

$$\Rightarrow \int \frac{5x+3}{\sqrt{x^2+4x+10}}dx = \int \frac{\frac{5}{2}(2x+4)-7}{\sqrt{x^2+4x+10}}dx$$

$$= \frac{5}{2}\int \frac{2x+4}{\sqrt{x^2+4x+10}}dx - 7\int \frac{1}{\sqrt{x^2+4x+10}}dx$$

Let $I_1 = \int \frac{2x+4}{\sqrt{x^2+4x+10}}dx$ and $I_2 = \int \frac{1}{\sqrt{x^2+4x+10}}dx$

$$\therefore \int \frac{5x+3}{\sqrt{x^2+4x+10}}dx = \frac{5}{2}I_1 - 7I_2 \qquad ...(1)$$

Then, $I_1 = \int \frac{2x+4}{\sqrt{x^2+4x+10}}dx$

Let $x^2 + 4x + 10 = t$

1/2

$$\therefore (2x+4)dx = dt$$

$$\Rightarrow I_1 = \int \frac{dt}{t} = 2\sqrt{t} = 2\sqrt{x^2+4x+10} \qquad ...(2)$$

$$I_2 = \int \frac{1}{\sqrt{x^2+4x+10}}dx$$

$$= \int \frac{1}{\sqrt{(x^2+4x+4)+6}}dx$$

1/2

$$= \int \frac{1}{(x+2)^2 + (\sqrt{6})^2}dx$$

$$= \log\left|(x+2)\sqrt{x^2+4x+10}\right| \qquad ...(3)$$

Any justified reason which will give deep sense of national integration will be awarded by one mark.

1

19.	$\int_0^{\frac{\pi}{2}} \log(\tan x + \cot x)\ \ dx = \int_0^{\frac{\pi}{2}} \log(\sin x/\cos x + \cos x/\sin x)\ \ dx$ $= \int_0^{\frac{\pi}{2}} \log(2/2\sin x.\cos x)\ \ dx$ $= \int_0^{\frac{\pi}{2}} (\log 2 - \log\sin 2x)\ \ dx$ $= \frac{\pi}{2}\log 2 - \int_0^{\frac{\pi}{2}} \log\sin 2x\ \ dx$ $= \frac{\pi}{2}\log 2 - \frac{1}{2}\int_0^{\pi} \log\sin t\ \ dt$ $= \frac{\pi}{2}\log 2 - \int_0^{\pi/2} \log\sin t\ \ dt$ $I = \frac{\pi}{2}\log 2 - I_1$ $I_1 = \int_0^{\pi/2} \log\sin t\ \ dt$ $= \int_0^{\pi/2} \log\sin(\pi/2\text{-}t)\ \ dt$ $2I_1 = \int_0^{\pi/2} \log\sin t + \log\cos t\ \ dt$ $I_1 = -\frac{\pi}{2}\log 2 \Rightarrow I = \pi\log 2$	**1** **1** **1** **1**
20.	Let $\vec{d} = d_1\hat{i} + d_2\hat{j} + d_3\hat{k}$. Since $\vec{d}$ is perpendicular to both $\vec{a}$ and $\vec{b}$, we have:	**½**

$\vec{d}\cdot\vec{a}=0$

$\Rightarrow d_1+4d_2+2d_3=0 \qquad \text{...(i)}$ **1**

And,

$\vec{d}\cdot\vec{b}=0$

$\Rightarrow 3d_1-2d_2+7d_3=0 \qquad \text{...(ii)}$

Also, it is given that:

$\vec{c}\cdot\vec{d}=15$

$\Rightarrow 2d_1-d_2+4d_3=15 \qquad \text{...(iii)}$

1

On solving (i), (ii), and (iii), we get:

$d_1=\frac{160}{3}, d_2=-\frac{5}{3} \text{ and } d_3=-\frac{70}{3}$

$\therefore \vec{d}=\frac{160}{3}\hat{i}-\frac{5}{3}\hat{j}-\frac{70}{3}\hat{k}=\frac{1}{3}\left(160\hat{i}-5\hat{j}-70\hat{k}\right)$

1

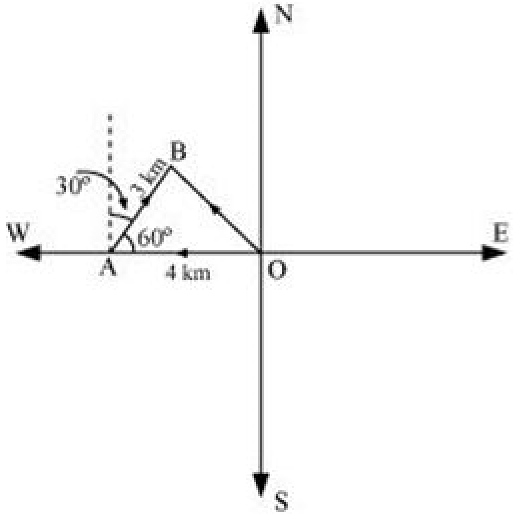
Hence, the required vector is $\frac{1}{3}\left(160\hat{i}-5\hat{j}-70\hat{k}\right)$.

Directionless student/youth is just like a stone on road any one can misguide him or her and drive them as per their own will, as they don't have any ambition or any aim moreover they always frustrated from the world and such type of people are the burden on society and the country. **1**

OR

Let O and B be the initial and final positions of the girl respectively.

Then, the girl's position can be shown as: **½(fig.)**



1

Now, we have:

$$\overrightarrow{OA} = -4\hat{i}$$

$$\overrightarrow{AB} = \hat{i}\left|\overrightarrow{AB}\right|\cos 60^\circ + \hat{j}\left|\overrightarrow{AB}\right|\sin 60^\circ$$

$$= \hat{i}3\times\frac{1}{2} + \hat{j}3\times\frac{\sqrt{3}}{2}$$

$$= \frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$

1/2

By the triangle law of vector addition, we have:

$$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$$

$$= \left(-4\hat{i}\right) + \left(\frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}\right)$$

$$= \left(-4 + \frac{3}{2}\right)\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$

$$= \left(\frac{-8+3}{2}\right)\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$

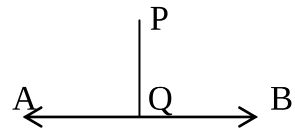
$$= \frac{-5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$$

1

Hence, the girl's displacement from her initial point of departure is

$$\frac{-5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}.$$

Respect the girls is the respect of the nation because if we respect the girls and ladies so we will respect the mother or sister or wife of someone's and this respect will lead the nation from hate speeches and battle we should never forget Mahabharata Battle and Demolishing of Ravan is just because Ladies Dropadi and Seeta respectively

21.	General point on the line (2r-1,3r+3,-r-2) Since PQ is perpendicular to AB This will give r = -15/14 Q = (-22/7, -3/14, -13/14) Now $PQ=\sqrt{\frac{531}{14}}$. 	**1/2 (Fig)** **½** **1** **1** **1**
22.	S= {HH,HT, T1,T2,…T6} E= {HT, T1,…T6} and F = {T5,T6} $E \cap F = \{T5, T6\}$ P(F)=3/4 P(E intersection F)=1/6 P(E|F)=2/9 **OR** Let P (E) = e and P (F) = f P (E $\cup$ F) – P (E $\cap$ F) =11/25 => e + f – 2ef =11/25… (1) P(E $\cap$F) =2/25 =>(1 – e) (1 – f) =2/25 =>1 – e – f + ef =2/25… (2) From (1) and (2) ef =12/25 and e + f =7/5 Solving, we get e =4/ 3, f=3/5 or e=3/ 5, f = 4/5	**1** **1** **1** **1** **1** **1** **1** **1**
23.	Let $A=\begin{bmatrix}1 & 3 & -2\\-3 & 0 & -5\\2 & 5 & 0\end{bmatrix}$ We know that $A = IA$ $\therefore \begin{bmatrix}1 & 3 & -2\\-3 & 0 & -5\\2 & 5 & 0\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}A$ Applying $R_2 \rightarrow R_2 + 3R_1$ and $R_3 \rightarrow R_3 - 2R_1$, we have:	**1** **1**

	$\begin{bmatrix}1 & 3 & -2\\0 & 9 & -11\\0 & -1 & 4\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\3 & 1 & 0\\-2 & 0 & 1\end{bmatrix}A$	
	Applying $R_1 \to R_1+3R_3$ and $R_2 \to R_2+8R_3$, we have:	1
	$\begin{bmatrix}1 & 0 & 10\\0 & 1 & 21\\0 & -1 & 4\end{bmatrix}=\begin{bmatrix}-5 & 0 & 3\\-13 & 1 & 8\\-2 & 0 & 1\end{bmatrix}A$	
	Applying $R_3 \to R_3+R_2$, we have:	1
	$\begin{bmatrix}1 & 0 & 10\\0 & 1 & 21\\0 & 0 & 25\end{bmatrix}=\begin{bmatrix}-5 & 0 & 3\\-13 & 1 & 8\\-15 & 1 & 9\end{bmatrix}A$	
	Applying $R_3 \to \frac{1}{25}R_3$, we have:	
	$\begin{bmatrix}1 & 0 & 10\\0 & 1 & 21\\0 & 0 & 1\end{bmatrix}=\begin{bmatrix}-5 & 0 & 3\\-13 & 1 & 8\\-\frac{3}{5} & \frac{1}{25} & \frac{9}{25}\end{bmatrix}A$	½
	Applying $R_1 \to R_1-10R_3$, and $R_2 \to R_2-21R_3$, we have:	½
	$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}=\begin{bmatrix}1 & -\frac{2}{5} & -\frac{3}{5}\\-\frac{2}{5} & \frac{4}{25} & \frac{11}{25}\\-\frac{3}{5} & \frac{1}{25} & \frac{9}{25}\end{bmatrix}A$	1

	$\therefore A^{-1} = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix}$	
24.	Revenue R = p x Profit= R - C $P = -1/3x^3 + 6x^2 - 11x - 50$ $dP/dx = -x^2 + 12x - 11$ $d^2P/dx^2 = -2x + 12$ *For* minima or maxima $dP/dx = 0$ *which* will give $x = 1, 11$ *at* $x = 1, d^2P/dx^2 = 10 > 0$ *Now* at $x = 11, d^2P/dx^2 = -10 < 0$ $\therefore$ *profit* will be maximum when $x = 11$ *and* max. *profit is* = ₹ 111.33 **OR** $\frac{dy}{dx} = \frac{-\sin(x+y)}{1+\sin(x+y)}$ $\sin(x+y) = 1$ since slope is -1/2 x+y = $n\pi + (-1)^n \frac{\pi}{2}, n \in Z$ $\Rightarrow y = 0 \ \forall \ n \in Z$ $\therefore$ points are $\left(\frac{-3\pi}{2}, 0\right)$ and $\left(\frac{\pi}{2}, 0\right)$ required equation of tangents are $2x + 4y + 3\pi = 0$ *and* $2x + 4y - \pi = 0$	**1** **1** **1** **1** **1** **1** **1** **1** **1** **1**

		1 **1**
25.	Correct fig(with proper stips and shading) Points of intersection (4,0)and (4,4) Required Area = area of region covered by parabola + area of the region covered by circle $=\int_0^4 ydx+\int_4^8 ydx$ $=2\int_0^4 \sqrt{x}dx+\int_4^8 \sqrt{4^2-(x-4)^2}dx$ $=\frac{4}{3}(8+3\pi)sq.units$	**½+1** **1** **½** **1** **1** **1**
26.	$\left[\frac{e^{-2\sqrt{y}}}{\sqrt{y}}-\frac{x}{\sqrt{y}}\right]\frac{dy}{dx}=1$; $(y\neq 0)$ $\frac{dx}{dy}+\frac{x}{\sqrt{y}}=\frac{e^{-2\sqrt{y}}}{\sqrt{y}}$ $IF=e^{\int\frac{1}{\sqrt{y}}dy}=e^{2\sqrt{y}}$ solution of given differential equation is given by $x.IF=\int IF.Qdy+c$ $xe^{2\sqrt{y}}=\int\frac{e^{-2\sqrt{y}}}{\sqrt{y}}e^{2\sqrt{y}}dy+c$ $xe^{2\sqrt{y}}=\int\frac{1}{\sqrt{y}}dy+c$ $xe^{2\sqrt{y}}=2\sqrt{y}+c$ **Now if** $y(1)=2$. **Then c =** $e^{2\sqrt{2}}-2\sqrt{2}$ $xe^{2\sqrt{y}}=2\sqrt{y}+e^{2\sqrt{2}}-2\sqrt{2}$ **OR**	**1** **1** **1** **1** **1** **1** 1 1 1

	Since this differential equation is homogeneous therefore Putting $y = vx$ $\frac{dy}{dx} = v + x\frac{dv}{dx}$ *we get*, $\frac{v\sin v - \cos v}{v\cos v}dv = \frac{2dx}{x}$ *Integrating* both sides we get log sec v - log v =2 log x + C $\frac{\sec v}{vx^2} = \pm C$ now replacing the value of y/x we get $\sec\left(\frac{y}{x}\right) = C\ x\ y.$ Which is the general solution of the given differential equation.	1 1 1
27.	Let A, E_1, and E_2 respectively denote the events that a person has a heart attack, the selected person followed the course of yoga and meditation, and the person adopted the drug prescription. $\therefore P(A) = 0.40$ $P(E_1) = P(E_2) = \frac{1}{2}$ $P(A\|E_1) = 0.40 \times 0.70 = 0.28$ $P(A\|E_2) = 0.40 \times 0.75 = 0.30$ Probability that the patient suffering a heart attack followed a course of meditation and yoga is given by P (E_1|A). $P(E_1\|A) = \frac{P(E_1)P(A\|E_1)}{P(E_1)P(A\|E_1) + P(E_2)P(A\|E_2)}$ $= \frac{\frac{1}{2}\times 0.28}{\frac{1}{2}\times 0.28 + \frac{1}{2}\times 0.30}$ $= \frac{14}{29}$ *Yoga increase oxygen combustion in mind and body and meditation increase the concentration of mind.Then we get peace in mind and healthy body. As we know a*	**1** **1/2** **1** **1** **1** **1+1/2**

	healthy body can possesses healthy mind.	

28.

Let the required line be parallel to vector $\vec{b}$ given by, **1**

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

The position vector of the point (1, 2, 3) is $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

The equation of line passing through (1, 2, 3) and parallel to $\vec{b}$ is given by, **1**

$$\vec{r} = \vec{a} + \lambda\vec{b}$$

$$\Rightarrow \vec{r}\left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right) \qquad ...(1)$$

The equations of the given planes are

$$\vec{r}\cdot\left(\hat{i} - \hat{j} + 2\hat{k}\right) = 5 \qquad ...(2)$$ **1**

$$\vec{r}\cdot\left(3\hat{i} + \hat{j} + \hat{k}\right) = 6 \qquad ...(3)$$

The line in equation (1) and plane in equation (2) are parallel. Therefore, the normal to the plane of equation (2) and the given line are perpendicular.

$$\Rightarrow \left(\hat{i} - \hat{j} + 2\hat{k}\right)\cdot\lambda\left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right) = 0$$ **1**

$$\Rightarrow \lambda\left(b_1 - b_2 + 2b_3\right) = 0$$

$$\Rightarrow b_1 - b_2 + 2b_3 = 0 \qquad ...(4)$$

Similarly, $\left(3\hat{i} + \hat{j} + \hat{k}\right)\cdot\lambda\left(b_1\hat{i} + b_2\hat{j} + b_3\hat{k}\right) = 0$

$$\Rightarrow \lambda\left(3b_1 + b_2 + b_3\right) = 0$$

$$\Rightarrow 3b_1 + b_2 + b_3 = 0 \qquad ...(5)$$ **1**

From equations (4) and (5), we obtain

$$\frac{b_1}{(-1)\times 1 - 1\times 2} = \frac{b_2}{2\times 3 - 1\times 1} = \frac{b_3}{1\times 1 - 3(-1)}$$

$$\Rightarrow \frac{b_1}{-3} = \frac{b_2}{5} = \frac{b_3}{4}$$

Therefore, the direction ratios of $\vec{b}$ are −3, 5, and 4.

$$\therefore \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k} = -3\hat{i} + 5\hat{j} + 4\hat{k}$$

<table>
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<td>29.</td>
<td>
Let the airline sell x tickets of executive class and y tickets of economy class.

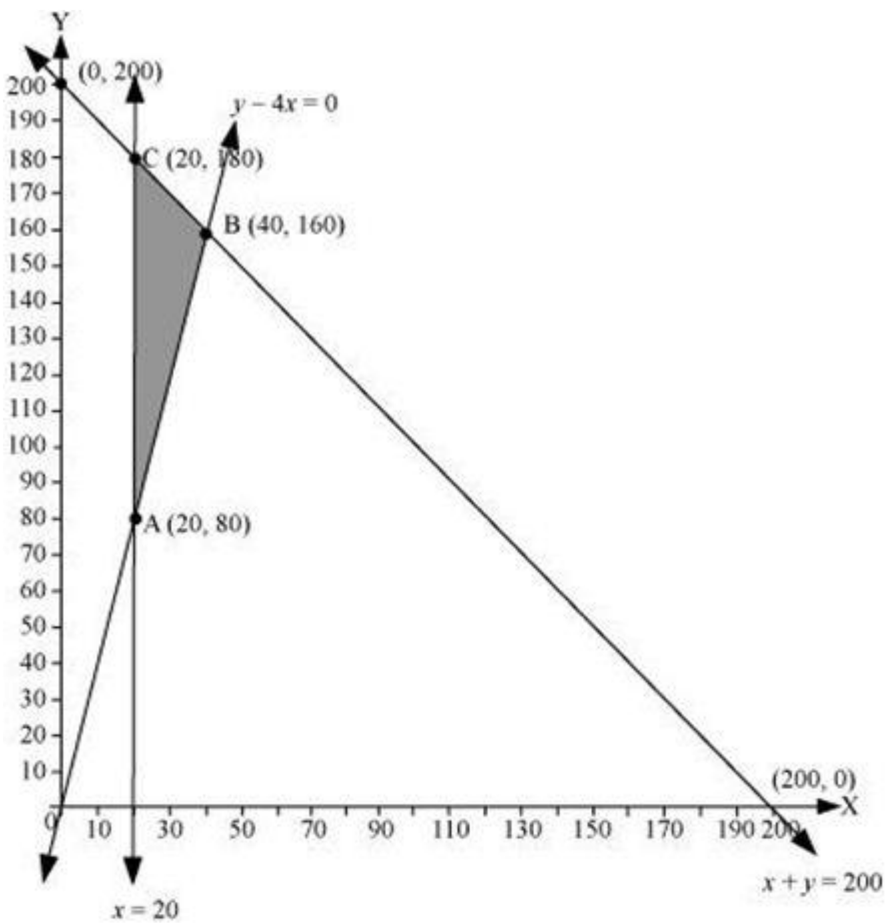
The mathematical formulation of the given problem is as follows.

Maximize $z = 1000x + 600y$ … (1)

subject to the constraints,

$x + y \leq 200$ ...(2)
$x \geq 20$ ...(3)
$y - 4x \geq 0$ ...(4)
$x, y \geq 0$ ...(5)

The feasible region determined by the constraints is as follows.



The corner points of the feasible region are A (20, 80), B (40, 160), and

C (20, 180).

The values of z at these corner points are as follows.
</td>
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1

2
Fig.</td>
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Corner point	$z = 1000x + 600y$	
A (20, 80)	68000	
B (40, 160)	136000	→ Maximum
C (20, 180)	128000	