

SAMPLE PAPER (2015)
XII
SUB:MATHEMATICS

Time: 3hrs

M.M:100

SECTION –A

1. Find the value of $\sin^{-1} \sin 10$.
2. Evaluate : $\int \cos^3 x \, dx$.
3. Give an example of a relation which is not a transitive relation .
4. Find a unit vector which is perpendicular to the vectors $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ & $\vec{b}=-\hat{i}+3\hat{j}+4\hat{k}$
5. Find the direction cosines of the line $\frac{x-2}{3} = \frac{3y-2}{7} = \frac{z-4}{6}$
6. For what value of x for which the matrix $A = \begin{bmatrix} 2x+3 & 6 \\ 5 & 8 \end{bmatrix}$ is singular?
7. If $A = \begin{bmatrix} 3 & 4 \\ 4 & 7 \end{bmatrix}$; then find the value of $|A^{-1}|$
8. Give the statement of Rolle's theorem.
9. Evaluate : $\int_{-3}^3 |x+1| \, dx$
10. Give the definition of scalar matrix with an example.

SECTION –B

11. If a, b, c are the lengths of sides of triangle then prove that area of triangle is given by

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)} \text{ , where s is the semi-perimeter of triangle.}$$

12. Evaluate: $\int \frac{3 \sin x + 2 \cos x}{4 \sin x + 3 \cos x} \, dx$

OR

Evaluate: $\int x \sin^{-1} \frac{1}{2} \sqrt{\frac{2a-x}{a}} \, dx$

13. Evaluate : $\int_0^{\pi/2} \frac{dx}{(a^2 \cos^2 x + b^2 \sin^2 x)^2}$

14. Prove that:

$$\cos[\tan^{-1}(\sin(\cot^{-1} x))] = \sqrt{\frac{1+x^2}{2+x^2}} .$$

OR

Prove that :

$$\tan^{-1} \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} = \frac{1}{2} \cos^{-1} \sqrt{x}$$

15. Solve the differential equation : $2x^2 \frac{dy}{dx} - 2xy + y^2 = 0$ given that $y(e) = e$.

16. Using properties of determinants , prove that :

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

17. Form the differential equation of the family of circles in the second quadrant and touching the co-ordinate axes.

18. Six dice are thrown 729 times .How many times do you expect at least three dice to show five or six?

19. Find the shortest distance and vector equation of line of shortest distance between the lines :

$$\frac{x}{2} = \frac{y}{-3} = \frac{z}{1} \text{ and } \frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$$

20 If function $f(x)$ given below is continuous function at $x = \frac{\pi}{2}$, then find the values of a and b:

$$f(x) = \begin{cases} \frac{1-\sin^3 x}{3\cos^2 x} & x < \frac{\pi}{2} \\ a & x = \frac{\pi}{2} \\ \frac{b(1-\sin x)}{(\pi-2x)^2} & x > \frac{\pi}{2} \end{cases}$$

21. If $y = \sin^{-1} [x^2 \sqrt{1-x^2} + x \sqrt{1-x^4}]$, then prove that $\frac{dy}{dx} = \frac{2x}{\sqrt{1-x^4}} + \frac{1}{\sqrt{1-x^2}}$

OR

If $x^p y^q = (x+y)^{p+q}$, then prove that $\frac{dy}{dx} = \frac{y}{x}$

22. Prove that the curve $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$ touches the straight line $\frac{x}{a} + \frac{y}{b} = 2$ at (a,b) for all values of $n \in \mathbb{N}$ at the point (a,b).

OR

Separate the interval $[0, \frac{\pi}{2}]$ into sub intervals in which $f(x) = \sin^4 x + \cos^4 x$ is increasing or decreasing.

SECTION-C

23. If P is a point on hypotenuse of a right triangle which is at a distance of a & b from the sides of triangle then prove that minimum length of hypotenuse is $(a^{3/2} + b^{3/2})^{2/3}$.

OR

A window is in the form of rectangle above which there is a semicircle. If the perimeter of the window is p cm. Show that the window will allow the maximum possible light only when the radius of the

semicircle is $\frac{p}{\pi+4}$ cm. .

24. Solve the following system of equations using matrix method:--

$$x+2y+3z=2$$

$$2x+3y+z=1$$

$$3x-y-4z=2$$

25. Using integration, find the area of triangle whose vertices are (2, 3), (4, 5), (6, 2).

26. Evaluate $\int_0^1 \frac{\sin^{-1}x}{x} dx$, using properties of definite integral.

OR

Using first principle evaluate : $\int_b^a \sin x dx$

27. In a test an examinee either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he makes a guess is $\frac{1}{3}$ and the probability that he copies the answer is $\frac{1}{6}$. The probability that his answer is correct, given that he copied it, is $\frac{1}{8}$. Find the probability that he knew the answer to the question, given that he correctly answered it.

28. A dealer wishes to purchase a number of fans and sewing machines. He has only Rs.5760.00 to invest and has space for at most 20 items. A fan costs him Rs. 360.00 and a sewing machine Rs.240.00. His expectation is that he can sell a fan at a profit of Rs.22.00 and a sewing machine at a profit of Rs.180.00. Assuming that he can sell all the items that he can buy, how should he invest his money in order to maximize his profit ?

29. Find the distance of the point (3, 4, 5) from the plane $x + y + z = 2$ measured parallel to the line $2x=y=z$.