

Model Test Paper-2015
CLASS–XII(CBSE-13)

Time–3 hrs.

MM–100

INSTRUCTIONS:

- i) All questions are compulsory.
- ii) The question paper consists of **29** questions divided into Three sections A, B and C. Section A comprises of **10** questions of **one mark** each, Section B comprises of **12** questions of **four marks** each, and Section C comprises of **7** questions of **six** marks each.
- iii) All questions in section A are to be answered in one word, one sentence or as per the exact requirements of the question.
- iv) There is no overall choice. However, internal choice has been provided in **4 questions of four marks each** and **2 questions of six marks each**. You have to attempt only one of the alternatives in all such questions.
- iv) Use of calculators is not permitted.

SECTIONS – A

1. If * be a binary operation defined by $a*b = 2a + b - 3$, then find $3*4$. [1]
2. Find the value of $\sin^{-1} \sin \left(\frac{3\pi}{5} \right)$ [1]
3. A square matrix A, of order 3, has $|A|=5$, find $|A.adj. A|$. [1]
4. Evaluate: $\begin{bmatrix} a + ib & c + id \\ -c - id & a - ib \end{bmatrix}$ [1]
5. For what value of x, is the matrix $\begin{bmatrix} 3 - 2x & x + 1 \\ 2 & 4 \end{bmatrix}$ is singular ? [1]
6. Prove that the function $-\frac{x^3}{3} + x^2 - x + \frac{3}{2}$ is decreasing in R. [1]
7. Evaluate: $\int \frac{3 \cos x}{2 \sin^2 x}$ [1]
8. If $\vec{a} = i\hat{a} + 2j\hat{b} - k\hat{c}$ and $\vec{b} = 3i\hat{a} + j\hat{b} - 5k\hat{c}$ then find a unit vector in the direction of $\vec{a} \times \vec{b}$ [1]
9. Find a vector of magnitude $\frac{5}{2}$ units which is parallel to the vector $3i\hat{a} + 4j\hat{b}$. [1]

10. If P(1, 5, 4) and Q(4, 1, -2), then find the direction ratio of $PQ \rightarrow$.

[1]

SECTIONS – B

11. Let $f, g : \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = |x|$ and $g(x) = [x]$, where $[x]$ denotes greatest integer less than or

equal to x . Then evaluate.
$$\frac{(g \circ f)\left(-\frac{5}{3}\right) - (f \circ g)\left(-\frac{5}{3}\right)}{(f \circ (g \circ f))\left(-\frac{5}{3}\right)}$$
 [4]

12. Express $\tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right)$, $-\frac{\pi}{2} < x < \frac{3\pi}{2}$ in the simplest form. [4]

OR, Solve $\tan^{-1}\frac{1-x}{1+x} \tan^{-1} x, x > 0$.

13. Prove that,
$$\begin{bmatrix} 1 + a^2 & ab & ac \\ ab & 1 + b^2 & bc \\ ca & cb & 1 + c^2 \end{bmatrix} = (1 + a^2 + b^2 + c^2).$$
 [4]

OR,

Prove that,
$$\begin{bmatrix} 2bc - a^2 & c^2 & b^2 \\ c^2 & 2ca - b^2 & a^2 \\ b^2 & a^2 & 2ab - c^2 \end{bmatrix} = (a^3 + b^3 + c^3 - 3abc)^2$$

14. Find the values of 'a' and 'b' such that the function defined by $f(x) = \begin{cases} 6 & \text{if } x \leq 2 \\ ax + b & 2 < x < 10 \\ 21 & \text{if } x \geq 10 \end{cases}$ is a continuous function. [4]

15. If $y^2 = 4ax$, prove that,
$$\left(\frac{d^2y}{dx^2}\right) \cdot \left(\frac{d^2x}{dy^2}\right) = -\frac{2a}{y^3}$$

[4]

OR,

Find the equation of the tangent to the curve $x^2 + 3y = 3$, which is parallel to the line $y - 4x + 5 = 0$.

16. If the radius of a sphere is measured as 9 cm with an error of 0.03 cm, then find the approximate error in calculating its volume.

[4]

OR,

Find the intervals in which the function f given by $f(x) = \sin x + \cos x$, $0 \leq x \leq 2\pi$ is strictly increasing or strictly decreasing.

17. Evaluate: $\int \frac{dx}{\sin(x-\alpha)\sin(x-\beta)}$

18. Evaluate: $\int_0^\pi \frac{x \cdot \sin x}{1 + \cos^2 x} dx$

19. Evaluate. $\int \frac{(x+x^3)^{\frac{1}{3}}}{x^4} dx$.

[4]

20. Let $a = 2i + k$, $b = i + j + k$ and $c = 4i - 3j + 7k$ be three vectors. Find a vector r which satisfies $r \times b = c \times b$ and $r \cdot a = 0$

[4]

21. Find the coordinates of the foot of the perpendicular drawn from the origin to the plane $2x + 3y + 4z - 12 = 0$. [4]

22. Find the probability distribution of number of heads in two tosses of a coin. [4]

SECTIONS – C

23. Using elementary operations, find the inverse of $A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$ if it exists.

[6]

24. A point P is given on the circumference of a circle of radius r . The chord QR is parallel to the tangent line at P . Find the maximum area of the triangle PQR .

[6]

25. Using integration, find the area of the circle $x^2 + y^2 = 16$, which is exterior to the parabola $y^2 = 6x$. [6]

OR, Using integration, find the area of the region bounded by the triangle whose vertices are $(1, 3)$, $(2, 5)$ and $(3, 4)$

26. Solve the differential equation : $(\tan^{-1}y - x) dy = (1 + y^2) dx$

[6]

27. Find the vector equation of the line passing through (1, 2, 3) and parallel to the $r \cdot (i^{\wedge} - j^{\wedge} + 2k^{\wedge}) = 5$ and $r \cdot (3i^{\wedge} + j^{\wedge} + k^{\wedge}) = 6$ planes [6]

OR, Find the perpendicular distance of the point (2, 3, 4) from the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$. Also find the coordinates of the foot of the perpendicular.

28. A company sells two different products A and B. The two products are produced in a common production process which has a total capacity of 500 man hours. It takes 5 hours to produce a unit of A and 3 hours to produce a unit of B. The demand in the market shows that the maximum number of units of A that can be sold is 70 and that of B is 125. Profit of each unit of A is ₹20 and on B is ₹15. How many units of A and B should be produced to maximize the profit? Form an L.P.P and solve it graphically. [6]

29. A manufacturer has three machine operators A, B and C. The first operator A produces 1% defective items, whereas the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of the time, B is on the job for 30% of the time and C is on the job for 20% of the time. A defective item is produced, what is the probability that was produced by A? [6]