SAMPLE PAPER-2015
Class XII

MATHEMATICS

Time Allowed: 3 hoursMaximum marks: 100

General Instructions:

1. All questions are compulsory.
2. The question paper consists of **29** questions divided into three sections **A**, **B** and **C**. Section A comprises of **10** questions of **one** mark each, Section B comprises of **12** questions of **four** marks each and Section C comprises of **7** questions of **six** marks each.
3. **All** questions in Section A are to be answered in **one** word, **one** sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in **4** questions of **four** marks each and **2** questions of **six** mark each. You have to attempt only **one** of the alternatives in all such questions.
5. Use of calculators is **not** permitted. You may ask for logarithmic tables, if required.

Section - A

1. If f: R→R is defined by f(x) = 3x + 2, define fof (x).
2. Write the principle value of $tan^{-1}(-1)$.
3. Write the values of x – y + z from the following equation : $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$
4. Write the order of the product matrix : $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} [2 \quad 3 \quad 4]$
5. If $\begin{vmatrix} x & x \\ 1 & x \end{vmatrix} = \begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix}$, write the positive value of x.
6. Evaluate : $\int \frac{(1+logx)^2}{x} dx$
7. Evaluate : $\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$
8. Write the position vector of the mid point of the vector joining the points P (2,3,4) and Q (4, 1, -2).
9. If $\vec{a}.\vec{a}.$ = 0 and $\vec{a}.\vec{b}.$ = 0, then what can be concluded about the vector $\vec{b}.$?
10. What are the direction cosines of a line, which makes equal angles with the co-ordinates axes?

Section – B

11. Consider f: $R_+ \rightarrow [4, \infty]$ given by f (x) = x^2 + 4 . Show that f is invertible with the inverse (f^{-1}) of f given by f^{-1}(y) = $\sqrt{y-4}$, where R_+ is the set of all non – negative real numbers.

12. Prove the following : $cot^{-1}\left[\frac{\sqrt{1+sinx}+\sqrt{1-sinx}}{\sqrt{1+sinx}-\sqrt{1-sinx}}\right] = \frac{x}{2}$, x ϵ (0, $\frac{\pi}{4}$)

Or

Find the value of $tan^{-1}\left(\frac{x}{y}\right)$ - $tan^{-1}\left(\frac{x-y}{x+y}\right)$

13. Using properties of determinants , prove that

$$\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2$$

14. Find the value of a for which the function f defined as f (x) = $\begin{cases} asin\frac{\pi(x+1),}{2} & , x \leq 0 \\ \frac{tan\ x-sinx}{x^3} & , x > 0 \end{cases}$

is continuous at x = 0

15. Find the points on the curve $x^2 + y^2 - 2x - 3 = 0$ at which the tangents are parallel to x – axis.

16. If x = a ($\theta - sin\theta$) , y = a (1 + cos θ) , find $\frac{d^2y}{dx^2}$

Or

Differentiate $sin^{-1}\left(2x\sqrt{1-x^2}\right)$ with respect to $cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

17. If $\sqrt{1-x^2}$ + $\sqrt{1-y^2}$ = a (x – y), prove that $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

18. Find the equation of plane passing through the point (1,2,1) and perpendicular to the line joining the points (1,4,2) and (2,3,5) . Also, find the perpendicular distance of the plane from the origin.

Or

The dot product of a vector with the vectors $\hat{\imath}$ - 3 $\hat{k}$, $\hat{\imath}$ - 2 $\hat{k}$ and $\hat{\imath}$ + $\hat{\jmath}$ + 4 $\hat{k}$ are 0 , 5 and 8 respectively . Find the vector.

19. Evaluate : $\int \frac{1}{cos(x-a)cos(x-b)}dx$

Or

Evaluate: $\int_0^{\frac{\pi}{2}} 2sinxcosx\ tan^{-1}(sinx)\ dx$

20. Solve the differential equation :

$cos^2 x \frac{dy}{dx} + y = tan\, x$

21. Find the mean number of heads in three tosses of a fair coin.
22. Find the shortest distance between the lines:

$\vec{r} = \hat{6i} + 2\hat{j} + 2\hat{k} + \lambda\,(\hat{\imath} - 2\,\hat{\jmath} + 2\,\hat{k})$ and

$\vec{r} = -4\hat{\imath} - \hat{k} + \mu\,(\hat{3i} - 2\,\hat{\jmath} - 2\,\hat{k})$

Section –C

23. Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equations :

x– y + 2z = 1 , 2y – 3z = 1 and 3x – 2y +4z = 2

Or

Using elementary transformations, find the inverse the matrix:

$$\begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$$

24. Using the method of integration , find the area of the region bounded by the lines :
2x + y = 4 , 3x – 2y = 6 and x – 3y + 5 = 0

25. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base.

Or

A window has the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12 m, find the dimensions of the rectangle that will produce the largest area of the window.

26. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.
27. Evaluate ; $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{tanx}}$

Or

Evaluate: $\int \frac{6x+7}{\sqrt{(x-5)(x-4)}}\,dx$

28. Find the equation of the plane passing through the line intersection of the planes
2x + y – z = 3 and 5x – 3y + 4z + 9 = 0 and parallel to the line $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-5}{5}$

29. Every gram of wheat provides 0.1 g of proteins and 0.25 g of carbohydrates. The corresponding values for rice are 0.05g and 0.5 g respectively. Wheat costs Rs. 4 per kg and rice Rs. 6 per kg. The minimum daily requirements of proteins and carbohydrates for an average child are 5 g and 200 g respectively. In what qualities should wheat and rice in the daily diet to provide minimum daily requirements of proteins and carbohydrates at minimum cost. Form a L.P.P. and solve it graphically.