

# SAMPLE PAPER-2015

## Class – XII

### Subject – Mathematics

#### SECTION A

Q1. Write the identity element for the binary operation  $*$  defined on the set  $R$  of real numbers by the rule  $a*b = \frac{3ab}{8}$ , for all  $a, b \in R$ .

Q2. If  $A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$

Write the order of  $AB$  and  $BA$ .

Q3. What is the principal value of  $\sin^{-1}\left(\sin\frac{5\pi}{6}\right) + \cos^{-1}\left(\cos\frac{\pi}{6}\right)$ .

Q4. Evaluate  $\begin{vmatrix} a & b & c \\ a+2x & b+2y & c+2z \\ x & y & z \end{vmatrix}$

Q5. If  $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ , evaluate  $F(0)$ .

Q6. Write the value of  $\int_0^{\frac{\pi}{2}} \left(\frac{3+5\cos x}{3+5\sin x}\right) dx$ .

Q7. Find the angle made by the vector  $\hat{j} - \hat{k}$  with the  $y$ -axis.

Q8. Find the value of  $k$  for which the lines

$$\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2} \text{ and } \frac{x-1}{3k} = \frac{y-1}{1} = \frac{6-z}{5}.$$

Are perpendicular to each other.

Q9. Write the value of  $\int e^{3\log x} \cdot x^4 dx$ .

Q10. Find  $|\vec{a} \times \vec{b}|$ , if  $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$  and  $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ .

#### SECTION B

Q11. Find the equation of the normals to the curve  $y = x^3 + 2x + 6$  which are parallel to the line  $x + 14y + 4 = 0$ .

Q12. If  $y = (\tan^{-1} x)^2$ , show that  $(x^2 + 1)y_2 + 2x(x^2 + 1)y_1 = 2$ .

Q13. Prove, using properties of determinants, that

$$\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x)$$

Q14. Let  $A$  and  $B$  be sets. Show that  $f: A \times B \rightarrow B \times A$  such that  $f(a, b) = (b, a)$  is a bijective function.

Q15. A doctor is to visit a patient. From the past experience, it is known that probabilities he will come by train, bus, scooter or by other means of transport are  $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$  and  $\frac{2}{5}$  respectively. The probabilities that he will be late are  $\frac{1}{4}, \frac{1}{3}$  and  $\frac{1}{12}$ , if

it comes by train, bus , scooter respectively ,but if comes by other means of transport, then he will not be late. When he arrives ,he is late. What is the probabilitythat he comes by train?

Q16.Evaluate the definite integral:

$$\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$$

Q17.If  $\tan^{-1} \frac{x-1}{x-2} + \tan^{-1} \frac{x+1}{x+2} = \frac{\pi}{4}$ , then find the value of x.

Q18. If  $x = a \left[ \cos t + \log \left| \tan \frac{t}{2} \right| \right]$  and  $y = a \sin t$  then find  $\frac{dy}{dx} \text{ at } t = \frac{\pi}{4}$ .

Q19. Find the distance of the point(-1,-5,-10) from the point of intersection of the line

$$\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$$

$$\text{And the plane } \vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$$

Q20. Let  $\vec{a} = 4\hat{i} + 5\hat{j} - \hat{k}$ ,  $\vec{b} = \hat{i} - 4\hat{j} + 5\hat{k}$  and  $\vec{c} = 3\hat{i} + \hat{j} - \hat{k}$ . Find a vector  $\vec{d}$  which is perpendicular to both  $\vec{a}$  and  $\vec{b}$ , and is such that  $\vec{d} \cdot \vec{c} = 21$ .

Q21.solve:  $x = dy - ydx = \sqrt{x^2 + y^2} dx$ .

Q22. Solve the following differential equation:

$$(1 + x^2) \frac{dy}{dx} + y = \tan^{-1} x$$

### SECTION C

23.If  $A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ , find  $(AB)^{-1}$ .

Q24.Find the image of the point (1,2,3) in the plane  $x + 2y + 4z = 38$ .

Q25.Find the area between the curves  $y=x^2$  and  $y = x$ .

Q26.An aero plane can carry a maximum of 200 passengers. A profit of Rs 1000 is made on each executive class ticket and a profit of Rs 600 is made on each economy class ticket .The airline reserve at least 20 seats for executive class.However, at least 4 times as many passengers prefer to travel by economy class than by executive class . Determine how many tickets of each type must be sold in order to maximize the profit for the airline. What is the maximum profit ?

Q27.Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is  $\frac{8}{27}$  of the volume of the sphere.

Q28.Two cards are drawn simultaneously (or successively without replacement) from A well shuffled pack of 52 cards. Find the mean, variance and standard deviation of the numbers of kings.

Q29.Evaluate:  $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$ .