

GENERAL INSTRUCTIONS :-

1. All question are compulsory.
2. The question paper consists of 29 questions divided into three sections A,B and C. Section–A comprises of 10 question of 1 mark each. Section – B comprises of 12 questions of 4 marks each and Section – C comprises of 7 questions of 6 marks each .
3. Question numbers 1 to 10 in Section – A are multiple choice questions where you are to select one correct option out of the given four.
4. There is no overall choice. However, internal choice has been provided in 4 question of four marks and 2 questions of six marks each. You have to attempt only one lf the alternatives in all such questions.
5. Use of calculator is not permitted.
6. Please check that this question paper contains 4 printed pages.
7. Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

Pre-Board Examination 2015 CLASS – XII CBSE MATHEMATICS

Time : 3 Hours

Maximum Marks: 100

SECTION-A

Q.1 Check whether the relation R in R defined byis transitive. $R=\{(a,b):a \leq b^2\}$ Ans =
not $(4,-3) \in R$ & $(-3,1) \in R \rightarrow (4,1) \notin R$

Q.2 Evaluate: $\int \frac{\cos 2x - \cos 2a}{\cos x - \cos a} dx$. Ans : $2\sin x + 2x \cos a$

Q.3 $A = \begin{bmatrix} 1 & 0 & 1 \\ k & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix}$, then write $|3A|$ and find the value of k, if A is singular matrix

ans : 0 (zero)

Q.4 The income I of Dr. Rastogi is given by $I(x) = Rs. (x^3 - 3x^2 + 5x)$. Can an insurance agent ensure him for the growth of his income? Ans : Show that $\frac{d}{dx} I(x) > 0$ for all x.

Q.5 Find the value of x if the area of Δ is 35 square cms with vertices $(x,4)$, $(2,-6)$ and $(5,4)$. Ans
 $x = -2$ or 12 .

Q.6 Evaluate: $\int [1 + 2\tan x(\tan x + \sec x)]^{1/2} dx$. Ans $\log_{10}(\sec x + \tan x) + \log_{10} \sec$

Q.7 The contentment obtained after eating x -units of a new dish at a trial function is given by the Function If the marginal contentment $C(x)=x^3+6x^2+5x+3$ is defined as rate of change of (x) with respect to the number of units consumed at an instant, then find the marginal contentment when three units of dish are consumed . ANS: 68 units

Q.8 If \vec{a} and \vec{b} are non-collinear vectors, find the value of x for which the vectors $\vec{a} + (2x + 1)\vec{b}$ and $(x - 2)\vec{a} + \vec{b}$ are collinear. Ans $x = 1/3$

Q.9 If $\vec{a} = \vec{b} + \vec{c}$, then is it true that $|\vec{a}| = |\vec{b}| + |\vec{c}|$? Justify your answer. Ans = not

Q.10 Find the perpendicular distance from $(2, 5, 6)$ on XY plane . Ans : 6 unit

SECTION-B

Q.11 Solve the following equation: $3\sin^{-1} \frac{2x}{1+x^2} - 4\cos^{-1} \frac{1-x^2}{1+x^2} + 2\tan^{-1} \frac{2x}{1-x^2} = \frac{\pi}{3}$. Ans $6\tan^{-1} x - 8\tan^{-1} x + 4\tan^{-1} x = \pi/3 \Rightarrow 2\tan^{-1} x = \pi/3 \Rightarrow \tan^{-1} x = \pi/6 \Rightarrow 2\tan^{-1} x = \pi/3$

OR

Solve for x : $2\tan^{-1}(\sin x) = \tan^{-1}(2 \sec x)$, $0 < x < \frac{\pi}{2}$. Ans. $x = \frac{\pi}{4} \in (0, \frac{\pi}{2})$

Q.12 If $f(x)$ and $g(x)$ be two invertible function defined as $f(x) = \frac{2x+1}{3x-5}$. be defined as Prove that $g(x) = \frac{3x+3}{7x-2}$ $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$. Ans: $(g \circ f)^{-1} x = \frac{15x-12}{8x-17} \rightarrow \frac{12+17y}{15-8y} f^{-1} = \frac{1+5x}{3x-2}$ & $g^{-1} = \frac{2x+3}{7x-3} \rightarrow f^{-1} \circ g^{-1} = \frac{12+7y}{15-8y}$

Q.13 Using the properties of determinants, prove the following:

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1-a^2+b^2)3.$$

Q.14 An air force plane is ascending vertically at the rate of 100 km/ h. If the radius of the earth is r km, how fast is the area of the earth , visible from the plane , increasing at 3 minutes after it started ascending ? Given that the visible area A at height h is given by $A = \frac{2\pi r^2 h}{r+h}$. Ans. $\frac{dA}{dt} = \frac{200\pi r^3}{(r+5)^2}$

Q.15 If $y = \sin(m \sin^{-1} x)$, prove that $(1-x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + m^2 y = 0$.

OR

If $X^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

Q.16 Two airships are moving in space along the following lines $\frac{x-3}{1} = \frac{5-y}{2} = \frac{z-7}{7}$ and $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$. An astronaut wants to move from one ship to another ship when two airships are closest. What is the least distance between the ships that he has to travel? Ans. Find S.D. $\rightarrow \sqrt{29}$ units.

Q.17 Find all the local maximum values and local minimum values of the function $f(x) = \sin 2x - x$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$. Ans. $F(x) = 0$ at $\mp \frac{\pi}{6}$. $F(x)$ is maximum at $x = \frac{\pi}{6}$ and maximum value is $f(\frac{\pi}{6}) = \frac{\sqrt{3}}{2} + \frac{\pi}{6}$ and $f(x)$ is minimum at $x = -\frac{\pi}{6}$ & minimum value is $f(-\frac{\pi}{6}) = -\frac{\sqrt{3}}{2} + \frac{\pi}{6}$

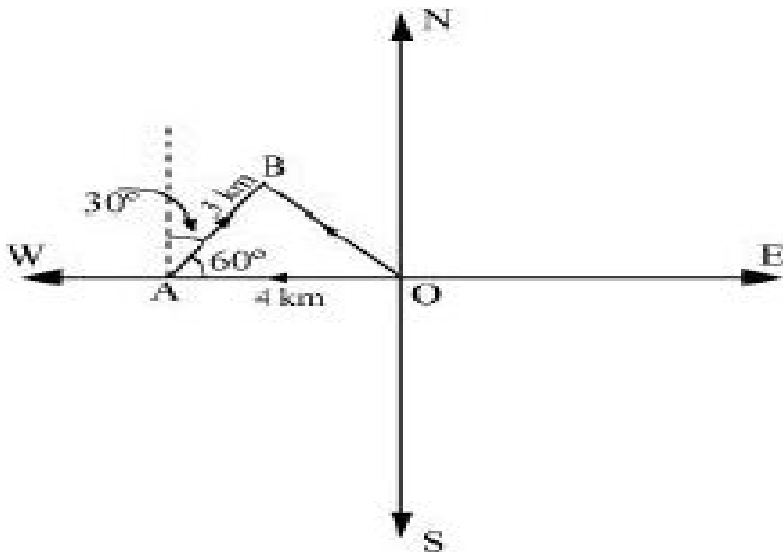
Q.18 Evaluate: $\int \frac{x}{x^3-1} dx$. Ans. $\frac{1}{3} \log(3x-1) - \frac{1}{6} \log(x^2+x+1) + \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{2x+1}{\sqrt{3}} \right)$

OR

Evaluate: $\int \frac{\sin 4x - 2}{1 - \cos 4x} e^{2x} dx$. Ans. $\frac{1}{3} \log e^{2x} \cot 2x$

Q.19 Solve the differential equation, $(1+y+x^2y)dx + (x+x^3)dy = 0$ where $y = 0$ when $x = 1$. Ans. $Yx = -\tan^{-1} x + \frac{\pi}{4}$

Q.20 A girl walks 5 km towards west, and then she walks 3 km in a direction 60° east of north and stops. Determine the girl's displacement from her initial point of departure. Respect the girl implies respect the nation comment on it. **ANS** : Let O and B be the initial and final positions of the girl respectively. Then, the girl's position can be shown as:



Q.21 Form the differential equation of the family of curve $y = ae^x + be^{2x} + ce^{3x}$; where a, b, c are some arbitrary constants. Ans. $Y_3 - 6Y_2 + 11Y_1 - 6Y = 0$

Q.22 The compressor used in refrigerators are manufactured by three different factories at Pune, Nasik and Nagpur. It is known that the Pune factory produces twice as many compressors as the Nasik one, which produces the same number as the Nagpur one (during

the same period). Experience also shows that 0.2 % of the compressors produced at Pune as well as at Nasik are defective and so are 0.4 % of those produced at Nagpur. A quality controller chooses a compressor and finds at a defective one. What is the probability that it was produced at Nasik factory? Ans 1/5

SECTION-C

Q.23 Three friends A, B and C visited a Super Market for purchasing fresh fruits. A purchased 1 kg apples, 3 kg grapes and 4 kg oranges and paid Rs. 800. B purchased 2 kg apples, 1 kg grapes and 2 kg orange and paid Rs. 500. White C paid Rs. 700 for 5 kg apples, 1 kg grapes and 1 kg oranges. Find the cost of each fruit per kg by using matrix method. Why are the fruits good for health? Ans. Let cost of each fruit be x, y, z respectively. Then solve the equations so formed by using matrix method. So, x = 100, y = 100, z = 100. Hence the cost of each fruit is 100 per kg. **Importance of fruits:** Fruits contain nutrients and vitamins which help our body in its proper growth and maintenance.

OR

Find the matrix P satisfying the matrix equation

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} P \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}. \text{Ans. } P = \begin{bmatrix} 25 & 15 \\ -37 & -22 \end{bmatrix}$$

Q.24 Reduce in symmetrical form , the equation of the line $x - y + 2z = 5$, $3x + y + z = 6$. ans.:

d . r . of line is -3 , 5 , 4 & point $\left(\frac{11}{4}, \frac{9}{4}, 0\right)$ Equation of required line $\frac{x-\frac{11}{4}}{-3} = \frac{y+\frac{9}{4}}{5} = \frac{z-0}{4}$ eq of line

$$\frac{4x-11}{-3} = \frac{4y+9}{5} = \frac{z}{1}$$

Q.25 Assume that the chances of a patient having a heart attack is 40%. It is also assumed that a meditation and yoga course reduce the risk of heart attack by 30% and prescription of certain drug reduces its chances by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga? Why Meditation and Yoga is necessary and sufficient thing for peace in mind and for good health. ANS

$$=PE_1) = P(E_2) = \frac{1}{2}; P(A/E_1) = \frac{40}{100} \times \left(1 - \frac{30}{100}\right) = \frac{28}{100}; P\left(\frac{A}{E_2}\right) = \frac{40}{100} \times \left(1 - \frac{25}{100}\right) = \frac{30}{100}$$

$= \frac{\frac{1}{2} \times \frac{28}{100}}{\frac{1}{2} \times \frac{30}{100} + \frac{1}{2} \times \frac{28}{100}} = \frac{28}{56} = \frac{14}{29}$ Ans. Yoga increase oxygen combustion in mind and body and meditation increase the concentration of mind. Then we get peace in mind and healthy body. As we know a healthy body can possess healthy mind.

Q.26 Draw the rough sketch of the region enclosed between the circles and $(x-2)^2+y^2=1$.

Using integration, find the area of the enclosed region.

DIAGRAM NO.2 IN 6285 WITH ANSWERS DIAGRAM OR PHOTOS FOLDER

Correct figure

Solving $x^2+y^2=4$ and $(x-2)^2+y^2=1$

Ans. We get $x=\frac{7}{4}$

$$\begin{aligned} \therefore \text{Required area} &= 2 \left[\int_{7/4}^2 \sqrt{4-x^2} dx + \int_1^{7/4} \sqrt{1-(x-2)^2} dx \right] = 2 \left[\frac{x}{2} \sqrt{4x^2+2} \sin^{-1} \frac{x}{2} \right]_{7/4}^2 \\ &+ \left[\frac{x-2}{2} \sqrt{1-(x-2)^2} + \frac{1}{2} \sin^{-1} (x-2) \right]_1^{7/4} \\ &= 2 \left[\pi - \frac{7\sqrt{15}}{8} - 2 \sin^{-1} \frac{7}{8} \right] + \left[\frac{1\sqrt{15}}{8} + \frac{1}{2} \sin^{-1} \left(-\frac{1}{4} \sin^{-1} \left[-\frac{1}{4} \right] + \frac{1}{2}, \frac{\pi}{2} \right) \right] = \frac{5\pi}{2} - \frac{\sqrt{15}}{2} - \sin^{-1} \left(\frac{1}{4} \right) - 4 \sin^{-1} \frac{7}{8} \end{aligned}$$

Parabola $y^2=ax$ Ans. $A_1 = \int_0^a \sqrt{ax} dx = \frac{2a^2}{3}$ & $A_2 = \int_0^1 \sqrt{a^2 - (x-a)^2} dx$ Or $\int_0^{2a} \sqrt{a^2 - (x-a)^2} dx = \frac{\pi a^2}{4} \therefore$

$$A_1 + A_2 = a^2 \frac{\pi}{4} + \frac{2}{3}$$

DIAGRAM NO.3 IN 6285 WITH ANSWERS DIAGRAM OR PHOTOS FOLDER

Q.27 Prove that all normal's to the curve $x = a \cos t + at \sin t$, $y = a \sin t - at \cos t$ are at a distance a from the origin. Ans; $dy/dx = \text{slope of tangent} = \tan t$; slope of normal = $-\cot t$ then equation of normal : $x \cos t + y \sin t = a$ & distance from origin is $= a$

Q.28 Evaluate: $\int_{-1/2}^{1/2} \left| x \cos \frac{\pi x}{2} \right| dx$. Ans. $\int_{-1/2}^0 -x \cos \frac{\pi x}{2} dx = \frac{1}{\pi\sqrt{2}} + \frac{4}{\pi^2\sqrt{2}} - \frac{4}{\pi^2} +$
 $\int_0^{1/2} x \cos \frac{\pi x}{2} dx = \frac{1}{\pi\sqrt{2}} - \frac{4}{\pi^2} = 2 \left(\frac{1}{\pi\sqrt{2}} + \frac{4}{\pi^2\sqrt{2}} - \frac{4}{\pi^2} \right)$

Q.29 An aero plane can carry a maximum of 200 passengers. A profit of 1000 is made on each executive class ticket and a profit of 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximize the profit for the airline. What is the maximum profit? How one should respect the hard earnmoney of parents/guardians in a best economical way. Ans: subject to the constraints,

$$X+y \leq 200 \quad \dots(2)$$

$$x \geq 20 \quad \dots(3)$$

$$y - 4x \geq 0 \quad \dots(4)$$

$x, y \geq 0$... (5) The feasible region determined by the constraints is as follows:

DIAGRAM NO.4 IN 6285 WITH ANSWERS DIAGRAM OR PHOTOS FOLDER

The corner points of the feasible region are A (20, 80), B (40, 160), and C (20, 180). The values of z at these cornerpoints are as follows.

Corner Point	Z= 1000x + 600y	
A(20,80)	68000	
B(40,160)	136000	---- Maximum
C(20,180)	128000	

The maximum value of z is 136000 at (40, 160). Thus, 40 tickets of executive class and 160 tickets of economy class should be sold to maximize the profit and the maximum profit is Rs 136000. Ans : Any justified answer which give clear idea of saving of hard earn money by parents will be awarded.