Sample Paper – 2015
Class – XII
Subject – MATHEMATICS

TIME : 3 hours **Max Marks:- 100**

General Instructions :

(i) All questions are compulsory.

(ii) The question paper consists of 29 questions divided into **3** sections **A, B** and **C**. Section A comprises of 10 questions of 1 mark each, Section B comprises of 12 questions of 4 marks each , Section C comprises of 7 questions of **6** marks each.

(iii) All questions in section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(iv) There is no overall choice. However internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

Section – A

Question numbers 1 to 10 carry 1 mark each.

1. Write $(f \circ g)(7)$, if f : R $\rightarrow$ R and g: R $\rightarrow$ R are given by $f(x) = x + 7$ and $g(x) = x - 7$.
2. What is the principal value of $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$.
3. If A = $\begin{bmatrix} 3 & 4 & 5 \end{bmatrix}$ and B = $\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$, find $B'A'$.
4. If A is a invertible matrix of order 3 and $|A| = 6$, then find $|\text{adj}(A)|$.
5. Find the slope of tangent to the curve $y = x^2 - 2$ at the point whose abscissa is 2.
6. Find the area of the region bounded by $y = |x - 1|$ between the lines $x = 0$ and $x = 1$.
7. Evaluate $\int \frac{\sec^2 \sqrt{x}}{\sqrt{x}}\, dx$.
8. If $2\sqrt{x} + 3\sqrt{y} = a$ then find $\frac{dy}{dx}$ at $\left(\frac{1}{4}, \frac{1}{4}\right)$.
9. Find the absolute maxima and absolute minima of the function $f(x) = x^3$, $x \in [-2, 2]$
10. Write the order and degree of the differential equation $\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 + 4y = 0$.

Section – B

Question numbers 11 to 22 carry 4 marks each.

11. Let Z be the set of integers and R be the relation defined on Z by R = { $(a, b) : a, b \in Z$, $|a - b|$ is divisible by 5}. Prove that R is an equivalence relation.

12. Solve for x : $\cos^{-1}\left(\frac{x^2-1}{x^2+1}\right) + \tan^{-1}\left(\frac{2x}{x^2-1}\right) = \frac{2\pi}{3}$.

Or

Prove that $\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right) + \tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right) = \frac{2b}{a}$.

13. Using properties of determinants, prove the following

$$\begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix} = 9b^2(a + b).$$

14. Verify Rolle's Theorem for the function $f(x) = \sin x + \cos x$, $x \in \left[0, \frac{\pi}{2}\right]$.

15. Express A = $\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}$ as the sum of a symmetric matrix and a skew symmetric matrix.

Or

Let A = $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$. Using principle of mathematical induction, prove that $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ for every positive integer n.

16. A particle moves along the curve $6y = x^3 + 5$. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate.

17. Evaluate : $\int_0^{\frac{\pi}{2}} \frac{1}{5+4\sin x}\,dx$.

18. Show that the function $f(x) = \begin{cases} 3x-2 & , \text{ if } 0 < x \le 1 \\ 2x^2 - x & , \text{ if } 1 < x \le 2 \\ 5x-4 & , \text{ if } x > 2 \end{cases}$ is continuous but not differentiable at at x = 2.

Or

If $x = 3 \sin t - \sin 3t$ and $y = 3 \cos t - \cos 3t$, then find $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{3}$..

19. Using differentials, find the approximate value of $\sqrt{0.037}$.

20. Evaluate : $\int \frac{1}{\sqrt{5-4x-2x^2}} dx$.

21. Find the particular solution of the differential equation $\frac{dy}{dx} + y \cot x = 4x \operatorname{cosec} x$, ($x \neq 0$), given that $y = 0$ when $x = \frac{\pi}{2}$.

Or

Solve the following differential equation $xy\frac{dy}{dx} + \sqrt{1+x^2+y^2+x^2y^2} = 0$.

22. Solve the linear programming problem graphically.

Minimize $z = x - 7y + 190$ subject to the constraints

$x + y \leq 8$; $x \leq 5$; $y \leq 5$; $x + y \geq 4$; $x \geq 0$; $y \geq 0$.

Section – C

Question numbers 23 to 29 carry 6 marks each.

23. Find the inverse of the following matrix using elementary operations. A = $\begin{bmatrix} 2 & 5 & 3 \\ 3 & 4 & 1 \\ 1 & 6 & 2 \end{bmatrix}$

24. Find the intervals in which the function $f(x) = (x - 1)(x - 2)^2$ is (*a*) strictly increasing (*b*) strictly decreasing.

Or

Show that the height of the cylinder of greatest volume which can be inscribed in a right circular cone of height *h* and semi-vertical angle α is one third that of the cone and greatest volume of the cylinder is $\frac{4}{27}\pi h^3 \tan^2 \alpha$.

25. Evaluate : $\int_0^{\pi} \frac{x}{a^2 \cos^2 x + b^2 \sin^2 x} dx$

26. An aeroplane can carry a maximum of 200 passengers. A profit of Rs 400 is made on each first class ticket and a profit of Rs 300 is made on each second class ticket. The airline reserves at least 20 seats for first class. However, at least 4 times as many passengers prefer to travel by second class than by the first class.

Determine how many tickets of each type must be sold in order to maximize the profit for the airline. Form an LPP and solve it graphically.

27. Find the area enclosed between the two circles: $x^2 + y^2 = 9$ and $(x - 3)^2 + y^2 = 9$.

Or

Find the maximum area of the isosceles triangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with its vertex at one end of the major axis.

28. Show that the general solution of the differential equation $\frac{dy}{dx} + \frac{y^2 + y + 1}{x^2 + x + 1} = 0$ is given by $(x + y + 1) = A(1 - x - y - 2xy)$, where A is a parameter.

29. A point on the hypotenuse of a right angled triangle is at distance a and b from the sides of the triangle. Show that the minimum length of the hypotenuse is $(a^{\frac{2}{3}} + b^{\frac{2}{3}})^{\frac{3}{2}}$.