General Instructions:

1. All question are compulsory.
2. The question paper consists of 29 questions divided into three sections A,B and C. Section – A comprises of 10 question of 1 mark each. Section – B comprises of 12 questions of 4 marks each and Section – C comprises of 7 questions of 6 marks each .
3. Question numbers 1 to 10 in Section – A are multiple choice questions where you are to select one correct option out of the given four.
4. There is no overall choice. However, internal choice has been provided in 4 question of four marks and 2 questions of six marks each. You have to attempt only one If the alternatives in all such questions.
5. Use of calculator is not permitted.
6. Please check that this question paper contains 5 printed pages.
7. Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

Pre-Board Examination 2015
CLASS – XII CBSE
MATHEMATICS

Time : 3 Hours **Maximum Marks : 100**

PART – A

Q.1 Evaluate: $x + y + z$ **if** $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$**.**Ans.0 Ans. $\begin{bmatrix} 0 & -1 & 1 \\ 2 & 0 & -2 \\ 5 & -2 & -3 \end{bmatrix}$

Q.2 If the vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $b = \hat{i} + 2\hat{j} + 3\hat{k}$ and $c = 3\hat{i} + \lambda\hat{j} + 5\hat{k}$ are coplanar, find the value of λ.Ans 2

Q.3 Find the vector equations of a line which passes through the point (1, 2, 3) and is parallel to the line $\frac{-x-2}{1} = \frac{y+3}{7} = \frac{2z-6}{3}$.

Q.4 A binary operation * on the set of rational number Q, is defined as a * b = a + b + ab. Check the operation * for associativity. Ans.* YES associative

Q.5 Evaluate: $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$. Ans. =1

Q.6 Find a unit vector in the direction of the resultant of vectors $\hat{i} + 2\hat{j} + 3\hat{k}$, □ $\hat{i} + 2\hat{j} + \hat{k}$ and $3\hat{i} + \hat{j}$.

Q.7 If $\sin^{-1} x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$, then the value of x.

Q.8 Find the projection of the vector $\hat{i}$ □ $2\hat{j} + \hat{k}$ on the vector $4\hat{i}$ □ $4\hat{j} + 7\hat{k}$. Ans. $\frac{19}{9}$

Q.9 If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $b = \hat{j} - \hat{k}$, find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \times \vec{c} =$ Ans. $\frac{5i+2j+2k}{3}$.

Q.10 Evaluate: $\int_0^{\pi/4} \sqrt{1 - \sin 2x\, dx}$. Ans. $\sqrt{2} - 1$

SECTION-B

Q.11 Solve the differential equation $x \frac{dy}{dx} = y(\log y - \log x + 1)$.

Solve the differential equation $\left(1+x^2\right) - \frac{4x^3}{3} = 0$. Find its particular solution, given that y = 0 when x = 0 .

Q.12 If a,b,c are non-zero numbers, show that: $\begin{bmatrix} -2a & a+b & a+c \\ b+a & -2b & b+c \\ c+a & c+b & -2c \end{bmatrix}$ =4(b+c)(c+a)(a+b)

Q.13 Evaluate: $\int_0^{\pi} \frac{x dx}{1-\cos a \sin x.}$ Ans. $\frac{\pi}{\sin a}(\pi - a)$

Q.14 Find the distance of the point (–1, –5, –10) from the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 5$. In your daily life which is most popular application of tossing a coin? Ans 13*unit* **Ans:** In daily life the most popular application of toss is in cricket. Before starting the game, a coin is tossed and the team who wins the toss decides that it will do batting or bowling first. There can be multiple answers to the value basedquestions. Students may have their own opinion about answering them, there is no specificsolution. Marks would be given for all sensible answers.

Q.15 Consider a function f: $R^+ \to [5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$, where R^+ is the set of all non-negative real numbers. Show that 'f' is invertible with $\int^{-1}(x) = \frac{\sqrt{x+6-1}}{3}$.

Q.16 If $(\sin^{-1} x^2\ )$ prove that :

Or

If $x = a(\theta$ □$- \sin\theta)$ & $y = a(1 - \cos\theta)$ find

$\frac{d^2y}{dx^2} at\theta = \frac{\pi}{2}$. Ans $\frac{d^2y}{dx^2} = \frac{1}{2} cosec^2 \frac{\theta}{2} X \frac{d\theta}{dx} = -\frac{1}{2} cosec^{2\frac{\theta}{2} X \frac{2}{a\sin\theta}} \to \left(\frac{d^2y}{dx^2}\right) = -\frac{1}{a}$

Q.17 The function f is given by f(x)= $\left\{\frac{1-sinx}{cos^2x}\ if \middle|\ x(\frac{\pi}{2}\right\}$

$$\left\{a \quad if \middle| x(\frac{\pi}{2}\right\}$$

$$\frac{b(1-sinx}{(\pi-2)2}\ if\ x)\ \frac{\pi}{2}$$

Find the values of a and b if f is continuous at x= $\frac{\pi}{2}$. Ans. A= $\frac{1}{2}; b = 4$.

Q.18 Form the differential equation corresponding to $y^2 = a(b - x^2)$, where *a* and *b* are arbitrary constants. Ans. $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right) = 0$

Q.19 The mean and variance of a binomial distribution are 4 and $\frac{4}{3}$ Ans. $P={}^{2}/_{3}, q = {}^{1}/_{3}, n = 6$

$p(x>,1)=1-\frac{1}{729}=\frac{728}{729}$

Q.20 Find the intervals in which the function *f* given by *f(x) = sin x – cos x,* $0 \le x \le 2\pi$ is (i) increasing, (ii) decreasing . Ans $\left(\frac{3\pi}{4}, \frac{7\pi}{4}\right) \downarrow, \uparrow \left(0, \frac{3\pi}{4}\right)$ & $\left(\frac{7\pi}{4}, 2\pi\right)$

or

Find the approximate value of f(5.001), where $f(x) = x^3 - 7x^2 + 15$. Ans -34 · 995

Q.21 Evaluate: $\int \frac{x \sin^{-1} x^2}{\sqrt{1-x^4}}$ Ans $\frac{1}{4}(\sin^{-1} x^2)^2$+c

Or

Evaluate: $\int \frac{2x-1}{(x-1)(x+2)(x-3)} dx$. Ans.-$\frac{1}{6}\log(x-1) - \frac{1}{3}\log(x+2) + \frac{1}{2}\log(x-3)$

Q.22 Solve for x: $\sin^{-1} x + \sin^{-1} 2x = \frac{\pi}{3}$. Ans x=$\sqrt{\frac{3}{28}}$

SECTION-C

Q.23 If $A=\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$, find A–1 and use it to solve the system of equations:: x + y + 2z = 0; x + 2y –z =9; x – 3y + 3z = -14. Ansx=1,y=3,z=-2; $A^{-1}=\frac{-1}{11}\begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix}$

Q.24 Evaluate: $\int \frac{dx}{x^{4+7x^2+25}}$ What are qualities of black colour? Ans=$\frac{1}{10\sqrt{17}}\tan^{-1}\left(\frac{x^2-5}{x\sqrt{17}}\right) - \frac{1}{20\sqrt{3}} log \left[\frac{x^2-\sqrt{3x+5}}{x^2+\sqrt{3x+5}}\right]$Ans. Black colour exhibits following qualities (a) it shows darkness. (b) it is morbid. There can be multiple answers to the value based questions. Students may have their own opinion about answering them, there is no specific solution. Marks would be given for all sensible answers/

Q.25 Find the point on the line $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{6}$ at a distance 3 from the point (1, 2,3).$(\frac{1}{7}, \frac{23}{7}, \frac{3}{7})$& $\left(\frac{13}{7}, \frac{5}{7}, \frac{39}{7}\right)$.

Find the vector and cartesian form of the equation of the plane containing the lines

r= (*i* + 2 *j* +3*k*)+λ (7*i* +3 *j* + 2*k*) and // to line *r* = (3*i* □ *j* + 3*k*)+ (2*i* + 2 *j* + 7*k*) Ans 17x-45y+8z+ 49 = 0&$\underset{r}{\rightarrow}$ (17i-45j+8k)+49=0

Q.26 Draw the rough sketch of $y^2 = x + 1$and $y^2 = -x + 1$and find the area enclosed by the two curves.

Ans. Required Area = $2\left(\int_0^{-1}\sqrt{(x+1)dx + \int_0^1 \sqrt{1-dx}}\right)=\frac{8}{3}unit^2$

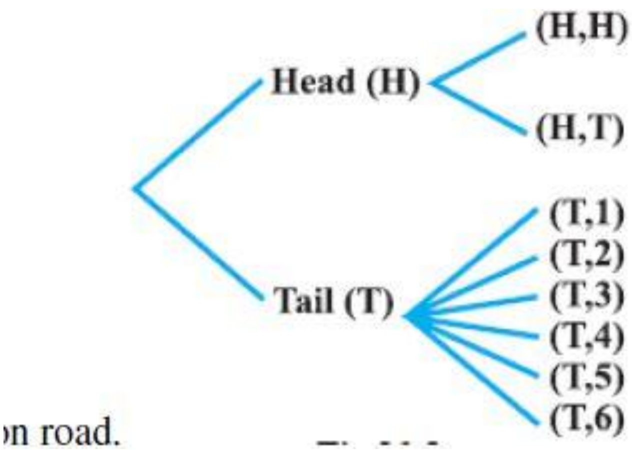
Q.27 Prove that the height and the radius of the base of an open cylinder of given surface area and maximum volume are equal.

Show that the semi-vertical angle or a cone of maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.

Q.28 A producer has 30 and 17 units of labour and capital respectively which he can use to produce two types of goods X and Y. To produce one ulnit of X, 3 units of capital and 2 units of labour are required and to produce one unit of Y, 3 units of labour and 1 unit of capital is required. If X and Y are priced at 100 and 120 respectively, how should the producer use his resources to maximize the total revenue ? From the LPP and solve it.

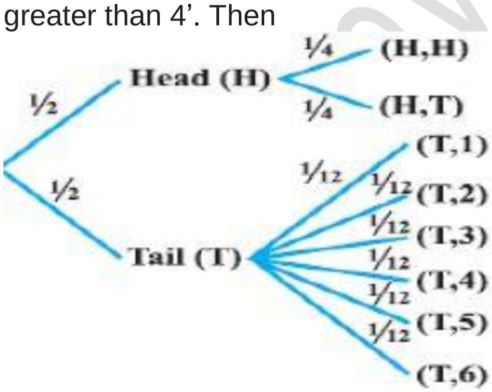
Ans z = 100x +120y;x,y≥0;2x+3y≤30,3x+y≤17,p☹0,10);$(\frac{17}{3},0)$ $(0,0)(3,8)$ $\max at$ $(3,8) = 1260$

Q.29 Consider the experiment of tossing a coin. If the coin shows head, toss it again but if it shows tail, then throw a die. Find the conditional probability of the event that 'the die shows a number greater than 4' given that 'there is at least one tail'. Give the points to be kept in mind while driving a vehicle on road.



Solution: The outcomes of the experiment can be represented in following diagrammatic manner called the 'tree diagram'.The sample space of the experiment may be described as S = {(H,H), (H,T), (T,1), (T,2), (T,3), (T,4), (T,5), (T,6)} where (H, H) denotes that both the tosses result into head and (T, *i*) denote the first toss result into a tail and the number *i* appeared on the die for *i* = 1,2,3,4,5,6. Thus, the probabilities assigned to the 8 elementary events (H, H), (H, T), (T, 1), (T, 2), (T, 3) (T, 4), (T, 5), (T, 6)

Let F be the event that 'there is at least one tail' and E be the event 'the die shows a number greater than 4'. Then



F={(H.T.).(T.1.).(T.2).(T.3.),(T.4.),(T.5.),(T.6.)}

E={(T,5),(T,6) and E O F= {(T,5), (T,6)}

Now P(F)=p({(H,T)})+p({(T,1)}+p({(T,2)})+p({(T,3)})+P({(T,4)})+P({(T,5)})+P({(T,6)})

=$\frac{1}{4}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}+\frac{1}{12}=\frac{3}{4}$ and P(E O E)=P({(T,5)})+P({(T,6)})=$\frac{1}{12}$+$\frac{1}{12}=\frac{1}{6}$

Hence P(EF) $\frac{P(E \ 0 \ F}{P(F)} = \frac{\frac{1}{6}}{\frac{3}{4}} = \frac{2}{9}$

ANS While driving a vehicle on road (a) we should always move on the hand side (b) traffic rules should be followed strictly. (c) we should drive the vehicle at normal speed. (d) we should obey the traffic signals. There can be multiple answers to the value based questions.Students may have their own opinion about answering them, there is no specific solution.Marks would be given for all sensible answers.

OR

The probability of a shooter hitting a target is 3 / 4 . How much minimum number of times must he/she fire so that the probability of hitting the target at least once is more than 0.99? Suggest necessary preparations to be made before going on a trip.

Solution: Let the shooter fire n times. Obviously, n fires are n Bemoulli trials. In each trial, p=probability of hitting the target $=\frac{3}{4}$ and q=probability of not hitting the target $=\frac{1}{4}$. Then $P(X=x)={}^{n}C_{x}q^{n-x}p^{x}={}^{n}c_{x}\left(\frac{1}{4}\right)^{n-x}\left(\frac{3}{4}\right)^{x}$ $={}^{n}C_{x}\frac{3^{x}}{4^{n}}$ Now, given that, P(hitting the target at least once)>0.99

i.e. $P(x \geq 1) > 0.99$

Therefore, 1-P(x=0)>0.99

Or $1-{}^{n}c_{x}\frac{1}{4n}<0.01$ i.e. $\frac{1}{4n}<0.01$

Or $4^{n} > \frac{1}{0.01} = 100$

The minimum value of n to satisfy the inequality (1) is 4. Thus the shooter must fire 4 times.

ANS preparations before going to trip are: (a) plan the trip (what to do and where to go)
(b) do not take too much of cash. Use debit cards and credit cards.
(c) check the weather forecast of the place.
(d) check the journey tickets for the detail and keep them with you. There can be multipleanswers to the value based questions. Students may have their own opinion about answering them, there is no specific solution. Marks would be given for all sensible answers.