CHAPTER 6

LINES AND ANGLES

(A) Main Concepts and Results

Complementary angles, Supplementary angles, Adjacent angles, Linear pair, Vertically opposite angles.

- If a ray stands on a line, then the adjacent angles so formed are supplementary and its converse,
- If two lines intersect, then vertically opposite angles are equal,
- If a transversal intersects two parallel lines, then
  - (i) corresponding angles are equal and conversely,
  - (ii) alternate interior angles are equal and conversely,
  - (iii) interior angles on the same side of the transversal are supplementary and conversely,
- Lines parallel to the same line are parallel to each other,
- Sum of the angles of a triangle is 180°,
- An exterior angle of a triangle is equal to the sum of the corresponding two interior opposite angles.

(B) Multiple Choice Questions

Write the correct answer:

Sample Question 1 : If two interior angles on the same side of a transversal intersecting two parallel lines are in the ratio 2 : 3, then the greater of the two angles is

(A) 54° (B) 108° (C) 120° (D) 136°

Solution : Answer (B)

EXERCISE 6.1

Write the correct answer in each of the following:

1. In Fig. 6.1, if AB || CD || EF, PQ || RS, ∠RQD = 25° and ∠CQP = 60°, then ∠QRS is equal to

(A) 85° (B) 135°

(C) 145° (D) 110°

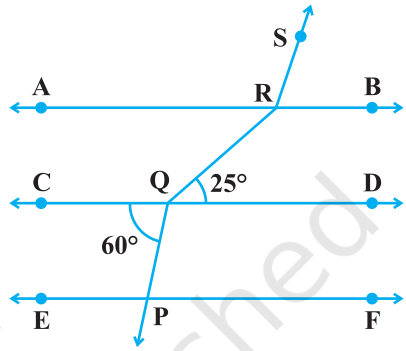

Fig. 6.1

2. If one angle of a triangle is equal to the sum of the other two angles, then the triangle is

(A) an isosceles triangle

(B) an obtuse triangle

(C) an equilateral triangle

(D) a right triangle

3. An exterior angle of a triangle is 105° and its two interior opposite angles are equal. Each of these equal angles is

(A) $37\frac{1}{2}^\circ$ (B) $52\frac{1}{2}^\circ$ (C) $72\frac{1}{2}^\circ$ (D) 75°

4. The angles of a triangle are in the ratio 5 : 3 : 7. The triangle is

(A) an acute angled triangle (B) an obtuse angled triangle

(C) a right triangle (D) an isosceles triangle

5. If one of the angles of a triangle is 130°, then the angle between the bisectors of the other two angles can be

(A) 50° (B) 65° (C) 145° (D) 155°

6. In Fig. 6.2, POQ is a line. The value of x is

(A) 20° (B) 25° (C) 30° (D) 35°

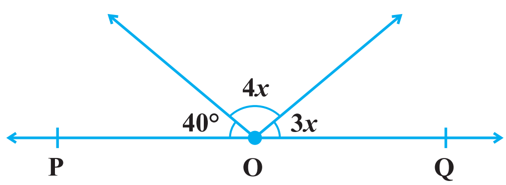

Fig. 6.2

7. In Fig. 6.3, if OP||RS, ∠OPQ = 110° and ∠QRS = 130°, then ∠ PQR is equal to

(A) 40° (B) 50° (C) 60° (D) 70°

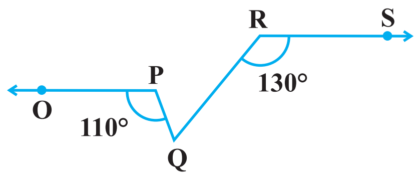

Fig. 6.3

8. Angles of a triangle are in the ratio 2 : 4 : 3. The smallest angle of the triangle is

(A) 60° (B) 40° (C) 80° (D) 20°

(C) Short Answer Questions with Reasoning

Sample Question 1 :

Let OA, OB, OC and OD are rays in the anticlockwise direction such that ∠ AOB = ∠COD = 100°, ∠BOC = 82° and ∠AOD = 78°. Is it true to say that AOC and BOD are lines?

Solution : AOC is not a line, because ∠ AOB + ∠ COB = 100° + 82° = 182°, which is not equal to 180°. Similarly, BOD is also not a line.

Sample Question 2 : A transversal intersects two lines in such a way that the two interior angles on the same side of the transversal are equal. Will the two lines always be parallel? Give reason for your answer.

Solution : In general, the two lines will not be parallel, because the sum of the two equal angles will not always be 180°. Lines will be parallel when each equal angle is equal to 90°.

EXERCISE 6.2

1. For what value of $x + y$ in Fig. 6.4 will ABC be a line? Justify your answer.

2. Can a triangle have all angles less than 60°? Give reason for your answer.

3. Can a triangle have two obtuse angles? Give reason for your answer.

4. How many triangles can be drawn having its angles as 45°, 64° and 72°? Give reason for your answer.

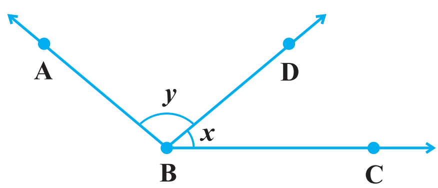

Fig. 6.4

5. How many triangles can be drawn having its angles as 53°, 64° and 63°? Give reason for your answer.

6. In Fig. 6.5, find the value of x for which the lines l and m are parallel.

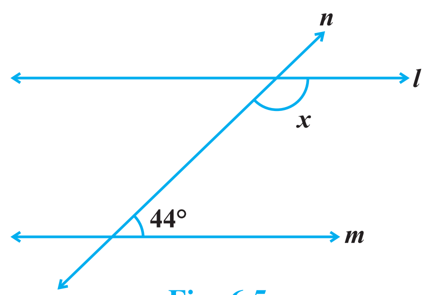

Fig. 6.5

7. Two adjacent angles are equal. Is it necessary that each of these angles will be a right angle? Justify your answer.

8. If one of the angles formed by two intersecting lines is a right angle, what can you say about the other three angles? Give reason for your answer.

9. In Fig.6.6, which of the two lines are parallel and why?

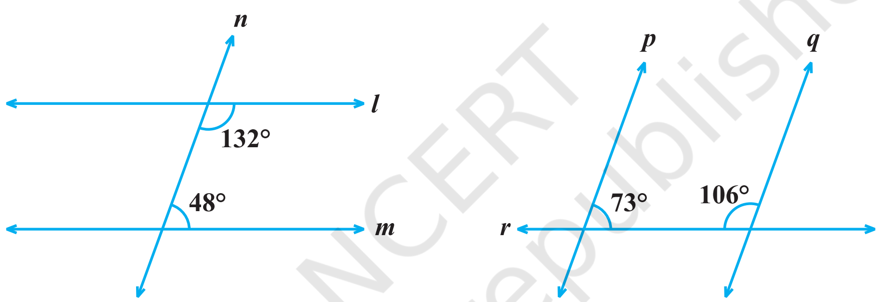

Fig. 6.6

10. Two lines l and m are perpendicular to the same line n. Are l and m perpendicular to each other? Give reason for your answer.

(D) Short Answer Questions

Sample Question 1 : In Fig. 6.7, AB, CD and EF are three lines concurrent at O. Find the value of y.

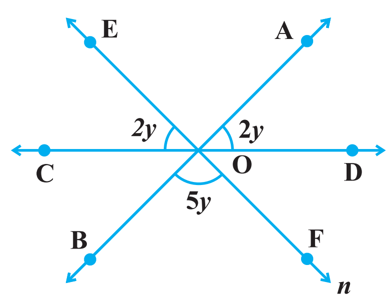

Fig. 6.7

Solution : $\angle AOE = \angle BOF = 5y$ (Vertically opposite angles)

Also,

$\angle COE + \angle AOE + \angle AOD = 180°$

So, $2y + 5y + 2y = 180°$

or, $9y = 180°$, which gives $y = 20°$.

Sample Question 2 : In Fig.6.8, $x = y$ and $a = b$. Prove that $l \parallel n$.

Solution: $x = y$ (Given)

Therefore, $l \parallel m$ (Corresponding angles) (1)

Also, $a = b$ (Given)

Therefore, $n \parallel m$ (Corresponding angles) (2)

From (1) and (2), $l \parallel n$ (Lines parallel to the same line)

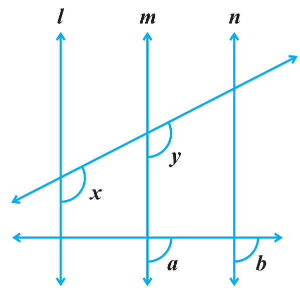

Fig. 6.8

EXERCISE 6.3

1. In Fig. 6.9, OD is the bisector of $\angle$AOC, OE is the bisector of $\angle$BOC and OD $\perp$ OE. Show that the points A, O and B are collinear.

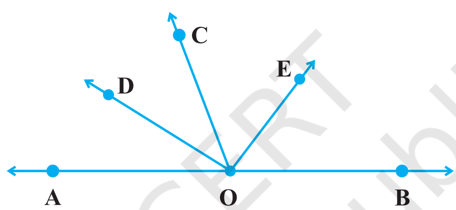

Fig. 6.9

2. In Fig. 6.10, $\angle 1 = 60°$ and $\angle 6 = 120°$. Show that the lines m and n are parallel.

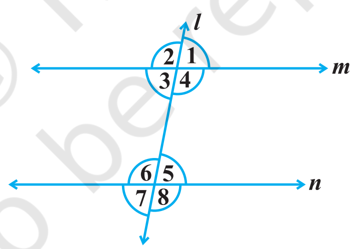

Fig. 6.10

3. AP and BQ are the bisectors of the two alternate interior angles formed by the intersection of a transversal t with parallel lines l and m (Fig. 6.11). Show that AP $\parallel$ BQ.

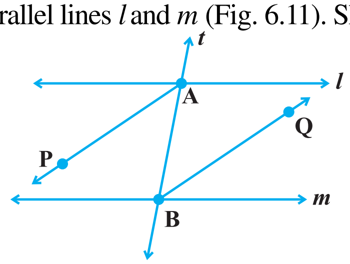

Fig. 6.11

4. If in Fig. 6.11, bisectors AP and BQ of the alternate interior angles are parallel, then show that $l \parallel m$.

5. In Fig. 6.12, BA || ED and BC || EF. Show that ∠ABC = ∠DEF
[Hint: Produce DE to intersect BC at P (say)].

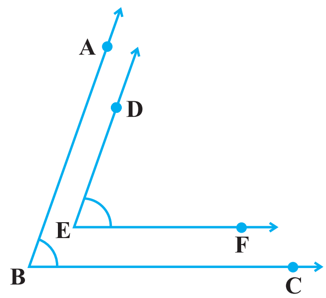

Fig. 6.12

6. In Fig. 6.13, BA || ED and BC || EF. Show that ∠ ABC + ∠ DEF = 180°

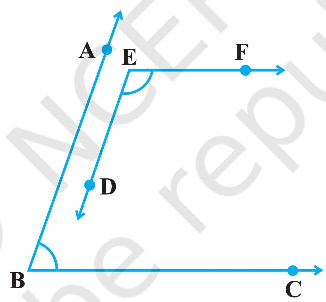

Fig. 6.13

7. In Fig. 6.14, DE || QR and AP and BP are bisectors of ∠ EAB and ∠ RBA, respectively. Find ∠APB.

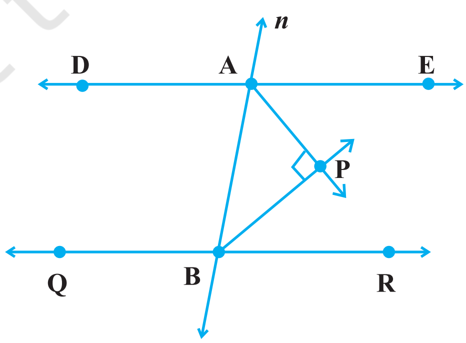

Fig. 6.14

8. The angles of a triangle are in the ratio 2 : 3 : 4. Find the angles of the triangle.

9. A triangle ABC is right angled at A. L is a point on BC such that AL $\perp$ BC. Prove that $\angle$ BAL = $\angle$ ACB.

10. Two lines are respectively perpendicular to two parallel lines. Show that they are parallel to each other.

(E) Long Answer Questions

Sample Question 1: In Fig. 6.15, *m* and *n* are two plane mirrors perpendicular to each other. Show that incident ray CA is parallel to reflected ray BD.

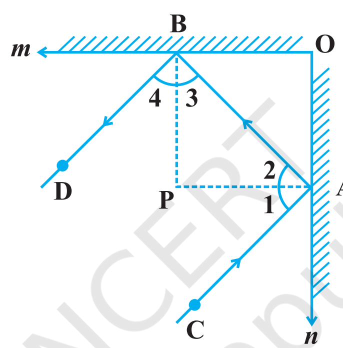

Fig. 6.15

Solution: Let normals at A and B meet at P.

As mirrors are perpendicular to each other, therefore, BP || OA and AP || OB.

So, BP $\perp$ PA, i.e., $\angle$ BPA = 90°

Therefore, $\angle 3 + \angle 2 = 90°$ (Angle sum property) (1)

Also, $\angle 1 = \angle 2$ and $\angle 4 = \angle 3$ (Angle of incidence = Angle of reflection)

Therefore, $\angle 1 + \angle 4 = 90°$ [From (1)] (2)

Adding (1) and (2), we have

$$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180°$$

i.e., $\angle CAB + \angle DBA = 180°$

Hence, CA || BD

Sample Question 2: Prove that the sum of the three angles of a triangle is 180°.

Solution: See proof of Theorem 6.7 in Class IX Mathematics Textbook.

Sample Question 3: Bisectors of angles B and C of a triangle ABC intersect each other at the point O. Prove that $\angle BOC = 90° + \frac{1}{2} \angle A$.

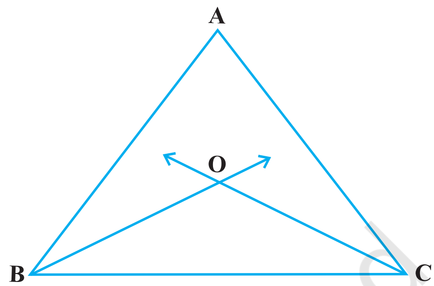

Fig. 6.16

Solution: Let us draw the figure as shown in Fig. 6.16

$\angle A + \angle ABC + \angle ACB = 180°$

(Angle sum property of a triangle)

Therefore, $\frac{1}{2} \angle A + \frac{1}{2} \angle ABC + \frac{1}{2} \angle ACB = \frac{1}{2} \times 180° = 90°$

i.e., $\frac{1}{2} \angle A + \angle OBC + \angle OCB = 90°$ (Since BO and CO are bisectors of $\angle B$ and $\angle C$) (1)

But $\angle BOC + \angle OBC + \angle OCB = 180°$ (Angle sum property) (2)

Subtracting (1) from (2), we have

$$\angle BOC + \angle OBC + \angle OCB - \frac{1}{2} \angle A - \angle OBC - \angle OCB = 180° - 90°$$

i.e., $\angle BOC = 90° + \frac{1}{2} \angle A$

EXERCISE 6.4

1. If two lines intersect, prove that the vertically opposite angles are equal.
2. Bisectors of interior $\angle B$ and exterior $\angle ACD$ of a Δ ABC intersect at the point T. Prove that

   $$\angle BTC = \frac{1}{2} \angle BAC.$$

3. A transversal intersects two parallel lines. Prove that the bisectors of any pair of corresponding angles so formed are parallel.

4. Prove that through a given point, we can draw only one perpendicular to a given line.

[**Hint:** Use proof by contradiction].

5. Prove that two lines that are respectively perpendicular to two intersecting lines intersect each other.

[**Hint:** Use proof by contradiction].

6. Prove that a triangle must have atleast two acute angles.

7. In Fig. 6.17, $\angle Q > \angle R$, PA is the bisector of $\angle QPR$ and PM $\perp$ QR. Prove that

$$\angle APM = \frac{1}{2}(\angle Q - \angle R).$$

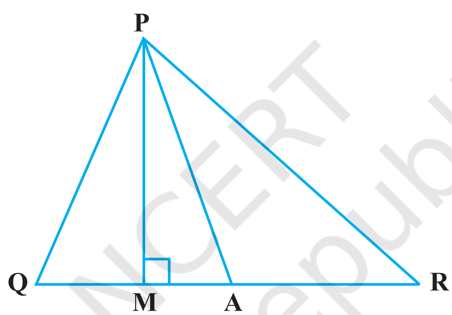

Fig. 6.17