UNIT 7

ALGEBRAIC EXPRESSIONS, IDENTITIES AND FACTORISATION

(A) Main Concepts and Results

(i) **Algebraic Expression**

- Terms are formed by the product of variables and constants, e.g. $-3xy$, $2xyz$, $5x^2$, etc.
- Terms are added to form expressions, e.g. $-2xy + 5x^2$.
- Expressions that contain exactly one, two and three terms are called monomials, binomials and trinomials, respectively.
- In general, any expression containing one or more terms with non-zero coefficients (and with variables having non-negative exponents) is called a polynomial.
- Like terms are formed from the same variables and the powers of these variables are also the same. But coefficients of like terms need not be the same.
- There are number of situations like finding the area of rectangle, triangle, etc. in which we need to multiply algebraic expressions.
- Multiplication of two algebraic expressions is again an algebraic expression.
- A monomial multiplied by a monomial always gives a monomial.
- While multiplying a polynomial by a monomial, we multiply every term in the polynomial by the monomial using the distributive law $a(b + c) = ab + ac$.

- In the multiplication of a polynomial by a binomial (or trinomial), we multiply term by term, i.e. every term of the polynomial is multiplied by every term in the binomial (or trinomial) using the distributive property.
- An identity is an equality, which is true for all values of its variables in the equality, i.e. an identity is a universal truth.
- An equation is true only for certain values of its variables.
- Some standard identities:

  (i) $(a + b)^2 = a^2 + 2ab + b^2$

  (ii) $(a - b)^2 = a^2 - 2ab + b^2$

  (iii) $(a + b)(a - b) = a^2 - b^2$

  (iv) $(x + a)(x + b) = x^2 + (a + b)x + ab$

(ii) **Factorisation**

- Representation of an algebraic expression as the product of two or more expressions is called factorisation. Each such expression is called a factor of the given algebraic expression.
- When we factorise an expression, we write it as a product of its factors. These factors may be numbers, algebraic (or literal) variables or algebraic expressions.

A **formula** is an equation stating a relationship between two or more variables. For example, the number of square units in the area (A) of a rectangle is equal to the number of units of length (l) multiplied by the number of units of width (w). Therefore, the formula for the area of a rectangle is A = lw.

Sometimes, you can evaluate a variable in a formula by using the given information.

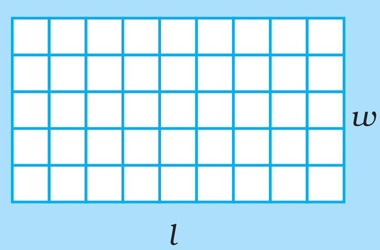
In the figure shown, the length is 9 units and the width is 5 units.

$$A = lw$$

$$A = 95$$

$$A = 45$$

The area is 45 square units or 45 units^2.



At other times, you must use your knowledge of equations to solve for a variable in a formula.

- An irreducible factor is a factor which cannot be expressed further as a product of factors. Such a factorisation is called an irreducible factorisation or complete factorisation.
- A factor which occurs in each term is called the common factor.
- The factorisation done by using the distributive law (property) is called the common factor method of factorisation.
- Sometimes, many of the expressions to be factorised are of the form or can be put in the form: $a^2 + 2ab + b^2$, $a^2 - 2ab + b^2$, $a^2 - b^2$ or $x^2 + (a + b)x + ab$. These expressions can be easily factorised using identities:

  $a^2 + 2ab + b^2 = (a + b)^2$

  $a^2 - 2ab + b^2 = (a - b)^2$

  $a^2 - b^2 = (a + b)(a - b)$

  $x^2 + (a + b)x + ab = (x + a)(x + b)$
- In the division of a polynomial by a monomial, we carry out the division by dividing each term of the polynomial by the monomial.
- In the division of a polynomial by a polynomial, we factorise both the polynomials and cancel their common factors.

(B) Solved Examples

In examples 1 to 4, there are four options given out of which one is correct. Write the correct answer.

Example 1 : Which is the like term as $24a^2bc$?

(a) $13 \times 8a \times 2b \times c \times a$ (b) $8 \times 3 \times a \times b \times c$

(c) $3 \times 8 \times a \times b \times c \times c$ (d) $3 \times 8 \times a \times b \times b \times c$

Solution : The correct answer is (a).

Example 2 : Which of the following is an identity?

(a) $(p + q)^2 = p^2 + q^2$ (b) $p^2 - q^2 = (p - q)^2$

(c) $p^2 - q^2 = p^2 + 2pq - q^2$ (d) $(p + q)^2 = p^2 + 2pq + q^2$

Solution : The correct answer is (d).

Example 3 : The irreducible factorisation of $3a^3 + 6a$ is

(a) $3a\,(a^2 + 2)$ (b) $3\,(a^3 + 2)$

(c) $a\,(3a^2 + 6)$ (d) $3 \times a \times a \times a + 2 \times 3 \times a$

Solution : The correct answer is (*a*).

Example 4 : $a\,(b + c) = ab + ac$ is

(a) commutative property (b) distributive property

(c) associative property (d) closure property

Solution : The correct answer is (*b*).

In examples 5 and 6, fill in the blanks to make the statements true.

Example 5 : The representation of an expression as the product of its factors is called __________.

Solution : Factorisation.

Example 6 : $(x + a)\,(x + b) = x^2 + (a + b)x +$ ________.

Solution : *ab*.

In examples 7 to 9, state whether the statements are true (T) or false (F).

Example 7 : An identity is true for all values of its variables.

Solution : True.

Example 8 : Common factor of x^2y and $-xy^2$ is xy.

Solution : True.

Example 9 : $(3x + 3x^2) \div 3x = 3x^2$

Solution : False.

Example 10 : Simplify (i) $-pqr\,(p^2 + q^2 + r^2)$

(ii) $(px + qy)\,(ax - by)$

Solution :

(i) $-pqr\,(p^2 + q^2 + r^2)$

$= -(pqr) \times p^2 - (pqr) \times q^2 - (pqr) \times r^2$

$= -p^3qr - pq^3r - pqr^3$

(ii) $(px + qy)\,(ax - by)$

$= px\,(ax - by) + qy\,(ax - by)$

$= apx^2 - pbxy + aqxy - qby^2$

Real-Life Math

Algebra in the Strongest Places : You might think that algebra is a topic found only in textbooks, but you can find algebra all around you – in some of the strongest places.

Did you know there is a relationship between the speed at which ants crawl and the air temperature? If you were to find some ants outside and time them as they crawled, you could actually estimate the temperature. Here is the algebraic equation that describes this relationship.

Celsius temperature → $t = 15s + 3$ ← **ant speed in centimetres per seconds**

There are many ordinary and extraordinary places where you will encounter algebra.

Think About it : What do you think is the speed of a typical ant?

Example 11 : Find the expansion of the following using suitable identity.

(i) $(3x + 7y)(3x - 7y)$ (ii) $\left(\frac{4x}{5}+\frac{y}{4}\right)\left(\frac{4x}{5}+\frac{3y}{4}\right)$

Solution : (i) $(3x + 7y)(3x - 7y)$

Since $(a + b)(a - b) = a^2 - b^2$, therefore

$(3x + 7y)(3x - 7y) = (3x)^2 - (7y)^2$

$= 9x^2 - 49y^2$

(ii) $\left(\frac{4x}{5}+\frac{y}{4}\right)\left(\frac{4x}{5}+\frac{3y}{4}\right)$

Since $(x + a)(x + b) = x^2 + (a + b)x + ab$, therefore

$$\left(\frac{4x}{5}+\frac{y}{4}\right)\left(\frac{4x}{5}+\frac{3y}{4}\right)$$

$$=\left(\frac{4x}{5}\right)^2+\left(\frac{y}{4}+\frac{3y}{4}\right)\times\frac{4x}{5}+\frac{y}{4}\times\frac{3y}{4}$$

$$=\left[\text{Here, } x=\frac{4x}{5},\ a=\frac{y}{4} \text{ and } b=\frac{3y}{4}\right]$$

$$=\frac{16x^2}{25}+\frac{4y}{4}\times\frac{4x}{5}+\frac{3y^2}{16}$$

$$=\frac{16x^2}{25}+\frac{4xy}{5}+\frac{3y^2}{16}$$

Example 12 : Factorise the following.

(i) $21x^2y^3 + 27x^3y^2$ (ii) $a^3 - 4a^2 + 12 - 3a$

(iii) $4x^2 - 20x + 25$ (iv) $\frac{y^2}{9} - 9$

(v) $x^4 - 256$

Solution : (i) $21x^2y^3 + 27x^3y^2$

$= 3 \times 7 \times x \times x \times y \times y \times y + 3 \times 3 \times 3 \times x \times x \times x \times y \times y$

$= 3 \times x \times x \times y \times y\ (7y + 9x)$ (Using $ab + ac = a\ (b + c)$)

$= 3x^2y^2\ (7y + 9x)$

(ii) $a^3 - 4a^2 + 12 - 3a$

$= a^2\ (a - 4) - 3a + 12$

$= a^2\ (a - 4) - 3\ (a - 4)$

$= (a - 4)\ (a^2 - 3)$

(iii) $4x^2 - 20x + 25$

$= (2x)^2 - 2 \times 2x \times 5 + (5)^2$

$= (2x - 5)^2$ (Since $a^2 - 2ab + b^2 = (a - b)^2$)

$= (2x - 5)\ (2x - 5)$

(iv) $\frac{y^2}{9} - 9$

$= \left(\frac{y}{3}\right)^2 - (3)^2$

If there are two numbers you don't know, that's not a problem. You can use two different variables, one for each unknown number.

In Words	Numbers
The sum of a and b	$a + b$
The product of v and w	$v \times w$, or vw
p is subtracted from $9q$	$q_q - p$

You can use expressions with two q–p (or more) variables to represent situations with more than one unknown quantity.

An equation involving variables can be true for all values of the variable – for example, y + y = 2y (this kind of equation is usually called an identity).
Or it can be true for only particular values of the variable – for example, 2y + 3 = 11, which is true only if y = 4.
Finding the values that make an equation true is called solving the equation.

$= \left(\frac{y}{3}+3\right)\left(\frac{y}{3}-3\right)$ (Since $a^2 - b^2 = (a + b)(a - b)$)

(v) $x^4 - 256$

$= (x^2)^2 - (16)^2$

$= (x^2 + 16)(x^2 - 16)$ (using $a^2 - b^2 = (a + b)(a - b)$)

$= (x^2 + 16)(x^2 - 4^2)$

$= (x^2 + 16)(x + 4)(x - 4)$ (using $a^2 - b^2 = (a + b)(a - b)$)

Example 13 : Evaluate using suitable identities.

(i) $(48)^2$ (ii) $18\ 1^2 - 19^2$

(iii) 497×505 (iv) 2.07×1.93

Solution : (i) $(48)^2$

$= (50 - 2)^2$

Since $(a - b)^2 = a^2 - 2ab + b^2$, therefore

$(50 - 2)^2 = (50)^2 - 2 \times 50 \times 2 + (2)^2$

$= 2500 - 200 + 4$

$= 2504 - 200$

$= 2304$

(ii) $181^2 - 19^2 = (181 - 19)(181 + 19)$

[using $a^2 - b^2 = (a - b)(a + b)$]

$= 162 \times 200$

$= 32400$

(iii) $497 \times 505 = (500 - 3)(500 + 5)$

$= 500^2 + (-3 + 5) \times 500 + (-3)(5)$ [using

$(x + a)(x + b) = x^2 + (a + b)x + ab$]

$= 250000 + 1000 - 15$

$= 250985$

(iv) $2.07 \times 1.93 = (2 + 0.07)(2 - 0.07)$

$= 2^2 - (0.07)^2$

$= 3.9951$

Example 14 : Verify that

$(3x + 5y)^2 - 30xy = 9x^2 + 25y^2$

Solution : L.H.S= $(3x + 5y)^2 - 30xy$

$= (3x)^2 + 2 \times 3x \times 5y + (5y)^2 - 30xy$

[Since $(a + b)^2 = a^2 + 2ab + b^2$]

$= 9x^2 + 30xy + 25y^2 - 30xy$

$= 9x^2 + 25y^2$

= R.H.S

Hence, verified.

Example 15 : Verify that $(11pq + 4q)^2 - (11pq - 4q)^2 = 176pq^2$

Solution : L.H.S. $(11pq + 4q)^2 - (11pq - 4q)^2$

$= (11pq + 4q + 11pq - 4q) \times (11pq + 4q - 11pq + 4q)$

[using $a^2 - b^2 = (a - b)(a + b)$, here $a = 11pq + 4q$ and $b = 11\ pq - 4q$]

$= (22pq)(8q)$

$= 176\ pq^2$

R.H.S. Hence Verified

To convert a Celsius temperature to a Fahrenheit temperature, find nine-fifths of the Celsius temperature and then add 32.	$F = \frac{9}{5}C + 32$

While the statement on the left may be easier to read and understand at first, the statement on the right has several advantages. It is shorter and easier to write, it shows clearly how the quantities – Celsius temperature and Fahrenheit temperature – are related, and it allows you to try different Celsius temperatures and compute their Fahrenheit equivalents.

Example 16 : The area of a rectangle is $x^2 + 12xy + 27y^2$ and its length is $(x + 9y)$. Find the breadth of the rectangle.

Solution : Breadth $= \frac{\text{Area}}{\text{Length}}$

$$= \frac{x^2 + 12xy + 27y^2}{(x + 9y)}$$

$$= \frac{x^2 + 9xy + 3xy + 27y^2}{(x + 9y)}$$

$$= \frac{x(x + 9y) + 3y(x + 9y)}{x + 9y}$$

$$= \frac{(x + 9y)\ (x + 3y)}{(x + 9y)}$$

$= (x + 3y)$

Example 17 : Divide $15\ (y + 3)\ (y^2 - 16)$ by $5\ (y^2 - y - 12)$.

Solution : Factorising $15\ (y + 3)\ (y^2 - 16)$,

we get $5 \times 3 \times (y + 3)\ (y - 4)\ (y + 4)$

On factorising $5\ (y^2 - y - 12)$, we get $5\ (y^2 - 4y + 3y - 12)$

$= 5\ [y\ (y - 4) + 3\ (y - 4)]$

$= 5\ (y - 4)\ (y + 3)$

Therefore, on dividing the first expression by the second expression, we get $\frac{15(y+3)\ (y^2 - 16)}{5(y^2 - y - 12)}$

$$= \frac{5 \times 3 \times (y + 3)(y - 4)(y + 4)}{5 \times (y - 4)(y + 3)}$$

$= 3\ (y + 4)$

Example 18 : By using suitable identity, evaluate $x^2 + \frac{1}{x^2}$, if $x + \frac{1}{x} = 5$.

Solution : Given that $x + \frac{1}{x} = 5$

So, $\left(x + \frac{1}{x}\right)^2 = 25$

Now, $\left(x + \frac{1}{x}\right)^2 = x^2 + 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2$ [Using identity $(a + b)^2 = a^2 + 2ab + b^2$, with $a = x$ and $b = \frac{1}{x}$]

$= x^2 + 2 + \left(\frac{1}{x^2}\right)$

$= x^2 + \left(\frac{1}{x^2}\right) + 2$

Since $\left(x+\frac{1}{x}\right)^2 = 25$, therefore $x^2 + \frac{1}{x^2} + 2 = 25$

or $x^2 + \frac{1}{x^2} = 25 - 2 = 23$

Example 19 : Find the value of $\frac{38^2-22^2}{16}$, using a suitable identity.

Solution : Since $a^2 - b^2 = (a + b)(a - b)$, therefore

$38^2 - 22^2 = (38 - 22)(38 + 22)$

$= 16 \times 60$

So, $\frac{38^2-22^2}{16} = \frac{16\times 60}{16}$

$= 60$

Example 20 : Find the value of x, if

$10000x = (9982)^2 - (18)^2$

Solution : R.H.S. $= (9982)^2 - (18)^2$

$= (9982 + 18)(9982 - 18)$ [Since $a^2 - b^2 = (a + b)(a - b)$]

$= (10000) \times (9964)$

L.H.S. $= (10000) \times x$

Comparing L.H.S. and R.H.S., we get

$10000x = 10000 \times 9964$

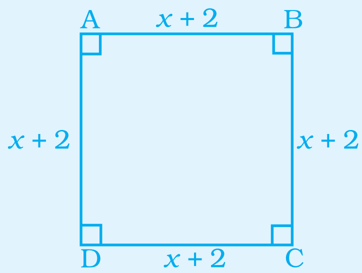
or $x = \frac{10000\times 9964}{10000} = 9964$

Think and Discuss

1. **Can you find** the reciprocal of $\frac{2}{11}\times\frac{5}{55}$?
2. **Can you compare** the ratio of this reciprocal with the earlier one?

Application on Problem Solving Strategy

Find each side of a figure given below, if its area is 64 cm².



Understand and Explore the problem

- What information is given in the question?

  AB = BC = DC = AD, and ∠A = ∠B = ∠C = ∠D = 90°
  Hence ABCD is a square.

- What are you trying to find?

  The value of one of the sides of the square ABCD.

- Is there any information that is not needed?

  No.

Make a Plan

- In a square all sides are equal, therefore, square of a side gives the area.

Solve

$(\text{Side})^2 = \text{Area}$

$\Rightarrow (x + 2)^2 = 64$

$\Rightarrow (x + 2)^2 = 8^2$

$\Rightarrow x + 2 = 8$

$\Rightarrow x = 8 - 2$

$\therefore x = 6$

$\therefore$ Side $= x + 2 = 6 + 2 = 8$ cm

Revise

- The above answer is verified by squaring the side and comparing the result with the given area.
  $\therefore (\text{Side})^2 = 8^2 = 64$ = given area.

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following:

1. The word *equivalent* contains the same root as the word *equal*. What do you think equivalent expressions are?
2. The word *simplify* means *make less complicated*. What do you think it means to simplify an expression?
3. The adjective *like* means *alike*. What do you suppose like terms are?
4. A system is a group of related objects. What do you think a system of equations is?

(C) Exercise

In questions 1 to 33, there are four options out of which one is correct. Write the correct answer.

1. The product of a monomial and a binomial is a

(a) monomial (b) binomial

(c) trinomial (d) none of these

2. In a polynomial, the exponents of the variables are always

(a) integers (b) positive integers

(c) non-negative integers (d) non-positive integers

3. Which of the following is correct?

(a) $(a - b)^2 = a^2 + 2ab - b^2$ (b) $(a - b)^2 = a^2 - 2ab + b^2$

(c) $(a - b)^2 = a^2 - b^2$ (d) $(a + b)^2 = a^2 + 2ab - b^2$

4. The sum of $-7pq$ and $2pq$ is

(a) $-9pq$ (b) $9pq$ (c) $5pq$ (d) $-5pq$

5. If we subtract $-3x^2y^2$ from x^2y^2, then we get

(a) $-4x^2y^2$ (b) $-2x^2y^2$ (c) $2x^2y^2$ (d) $4x^2y^2$

6. Like term as $4m^3n^2$ is

(a) $4m^2n^2$ (b) $-6m^3n^2$ (c) $6pm^3n^2$ (d) $4m^3n$

7. Which of the following is a binomial?

(a) $7 \times a + a$ (b) $6a^2 + 7b + 2c$

(c) $4a \times 3b \times 2c$ (d) $6\ (a^2 + b)$

8. Sum of $a - b + ab$, $b + c - bc$ and $c - a - ac$ is

(a) $2c + ab - ac - bc$ (b) $2c - ab - ac - bc$

(c) $2c + ab + ac + bc$ (d) $2c - ab + ac + bc$

9. Product of the following monomials $4p$, $-\ 7q^3$, $-7pq$ is

(a) $196\ p^2q^4$ (b) $196\ pq^4$ (c) $-\ 196\ p^2q^4$ (d) $196\ p^2q^3$

10. Area of a rectangle with length $4ab$ and breadth $6b^2$ is

(a) $24a^2b^2$ (b) $24ab^3$ (c) $24ab^2$ (d) $24ab$

11. Volume of a rectangular box (cuboid) with length = $2ab$, breadth = $3ac$ and height = $2ac$ is

(a) $12a^3bc^2$ (b) $12a^3bc$ (c) $12a^2bc$

(d) $2ab + 3ac + 2ac$

The five figures form a pattern.

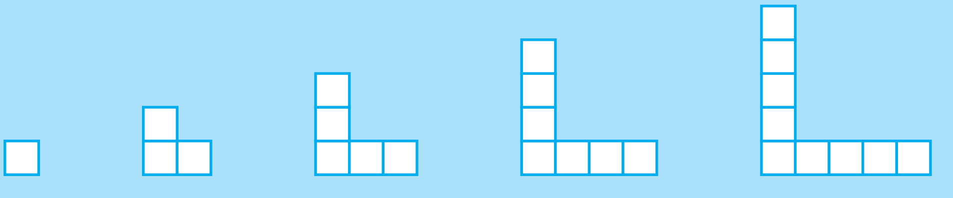


Figure 1 Figure 2 Figure 3 Figure 4 Figure 5

1. Copy and complete the table to find the perimeter of each figure. Each side of each individual square is 1 unit.

Figure	1	2	3	4	5
Perimeter					

2. Without drawing a picture, describe what the sixth figure will look like and predict its perimeter.

3. If you continue this pattern, what will be the perimeter of the 35th figure?

4. Explain how the perimeter of each figure is related to its figure number.

5. Using the variables n for the figure number and P for the perimeter, write an equation for the relationship in Question 4.

12. Product of $6a^2 - 7b + 5ab$ and $2ab$ is

(a) $12a^3b - 14ab^2 + 10ab$ (b) $12a^3b - 14ab^2 + 10a^2b^2$

(c) $6a^2 - 7b + 7ab$ (d) $12a^2b - 7ab^2 + 10ab$

13. Square of $3x - 4y$ is

(a) $9x^2 - 16y^2$ (b) $6x^2 - 8y^2$

(c) $9x^2 + 16y^2 + 24xy$ (d) $9x^2 + 16y^2 - 24xy$

14. Which of the following are like terms?

(a) $5xyz^2, -3xy^2z$ (b) $-5xyz^2, 7xyz^2$

(c) $5xyz^2, 5x^2yz$ (d) $5xyz^2, x^2y^2z^2$

15. Coefficient of y in the term $\frac{-y}{3}$ is

(a) -1 (b) -3 (c) $\frac{-1}{3}$ (d) $\frac{1}{3}$

16. $a^2 - b^2$ is equal to

(a) $(a - b)^2$ (b) $(a - b)(a - b)$

(c) $(a + b)(a - b)$ (d) $(a + b)(a + b)$

17. Common factor of $17abc$, $34ab^2$, $51a^2b$ is

(a) $17abc$ (b) $17ab$ (c) $17ac$ (d) $17a^2b^2c$

18. Square of $9x - 7xy$ is

(a) $81x^2 + 49x^2y^2$ (b) $81x^2 - 49x^2y^2$

(c) $81x^2 + 49x^2y^2 - 126x^2y$ (d) $81x^2 + 49x^2y^2 - 63x^2y$

19. Factorised form of $23xy - 46x + 54y - 108$ is

(a) $(23x + 54)(y - 2)$ (b) $(23x + 54y)(y - 2)$

(c) $(23xy + 54y)(-46x - 108)$ (d) $(23x + 54)(y + 2)$

20. Factorised form of $r^2 - 10r + 21$ is

(a) $(r - 1)(r - 4)$ (b) $(r - 7)(r - 3)$

(c) $(r - 7)(r + 3)$ (d) $(r + 7)(r + 3)$

21. Factorised form of $p^2 - 17p - 38$ is

(a) $(p - 19)(p + 2)$ (b) $(p - 19)(p - 2)$

(c) $(p + 19)(p + 2)$ (d) $(p + 19)(p - 2)$

22. On dividing $57p^2qr$ by $114pq$, we get

(a) $\frac{1}{4}pr$ (b) $\frac{3}{4}pr$ (c) $\frac{1}{2}pr$ (d) $2pr$

23. On dividing $p(4p^2 - 16)$ by $4p(p - 2)$, we get

(a) $2p + 4$ (b) $2p - 4$ (c) $p + 2$ (d) $p - 2$

24. The common factor of $3ab$ and $2cd$ is

(a) 1 (b) – 1 (c) a (d) c

25. An irreducible factor of $24x^2y^2$ is

(a) x^2 (b) y^2 (c) x (d) $24x$

26. Number of factors of $(a + b)^2$ is

(a) 4 (b) 3 (c) 2 (d) 1

27. The factorised form of $3x - 24$ is

(a) $3x \times 24$ (b) $3(x - 8)$ (c) $24(x - 3)$ (d) $3(x - 12)$

28. The factors of $x^2 - 4$ are

(a) $(x - 2), (x - 2)$ (b) $(x + 2), (x - 2)$

(c) $(x + 2), (x + 2)$ (d) $(x - 4), (x - 4)$

29. The value of $(-27x^2y) \div (-9xy)$ is

(a) $3xy$ (b) $-3xy$ (c) $-3x$ (d) $3x$

30. The value of $(2x^2 + 4) \div 2$ is

(a) $2x^2 + 2$ (b) $x^2 + 2$ (c) $x^2 + 4$ (d) $2x^2 + 4$

31. The value of $(3x^3 + 9x^2 + 27x) \div 3x$ is

(a) $x^2 + 9 + 27x$ (b) $3x^3 + 3x^2 + 27x$

(c) $3x^3 + 9x^2 + 9$ (d) $x^2 + 3x + 9$

32. The value of $(a + b)^2 + (a - b)^2$ is

(a) $2a + 2b$ (b) $2a - 2b$ (c) $2a^2 + 2b^2$ (d) $2a^2 - 2b^2$

33. The value of $(a + b)^2 - (a - b)^2$ is

(a) $4ab$ (b) $-4ab$ (c) $2a^2 + 2b^2$ (d) $2a^2 - 2b^2$

In questions 34 to 58, fill in the blanks to make the statements true:

34. The product of two terms with like signs is a ______ term.

35. The product of two terms with unlike signs is a ______ term.

36. $a(b + c) = ax$ ____ $\times$ ax _____.

37. $(a - b)$ _________ $= a^2 - 2ab + b^2$

38. $a^2 - b^2 = (a + b)$ __________.

39. $(a - b)^2 +$ ____________ $= a^2 - b^2$

40. $(a + b)^2 - 2ab =$ ___________ + ____________

41. $(x + a)(x + b) = x^2 + (a + b)x +$ ________.

42. The product of two polynomials is a ________.

43. Common factor of $ax^2 + bx$ is __________.

44. Factorised form of $18mn + 10mnp$ is ________.

45. Factorised form of $4y^2 - 12y + 9$ is ________.

46. $38x^3y^2z \div 19xy^2$ is equal to _________.

47. Volume of a rectangular box with length $2x$, breadth $3y$ and height $4z$ is _________.

48. $67^2 - 37^2 = (67 - 37) \times$ ________ = _________.

49. $103^2 - 102^2 =$ ________ $\times (103 - 102) =$ _________.

50. Area of a rectangular plot with sides $4x^2$ and $3y^2$ is __________.

51. Volume of a rectangular box with $l = b = h = 2x$ is _________.

52. The coefficient in $-37abc$ is __________.

53. Number of terms in the expression $a^2 + bc \times d$ is ________.

54. The sum of areas of two squares with sides $4a$ and $4b$ is _______.

55. The common factor method of factorisation for a polynomial is based on ___________ property.

56. The side of the square of area $9y^2$ is __________.

57. On simplification $\frac{3x+3}{3} =$ _________

58. The factorisation of $2x + 4y$ is __________.

In questions 59 to 80, state whether the statements are True (T) or False (F):

59. $(a + b)^2 = a^2 + b^2$

60. $(a - b)^2 = a^2 - b^2$

61. $(a + b)\ (a - b) = a^2 - b^2$

62. The product of two negative terms is a negative term.

63. The product of one negative and one positive term is a negative term.

64. The coefficient of the term $-\ 6x^2y^2$ is $-\ 6$.

65. $p^2q + q^2r + r^2q$ is a binomial.

66. The factors of $a^2 - 2ab + b^2$ are $(a + b)$ and $(a + b)$.

67. h is a factor of $2\pi\ (h + r)$.

68. Some of the factors of $\frac{n^2}{2}+\frac{n}{2}$ are $\frac{1}{2}, n$ and $(n+1)$.

69. An equation is true for all values of its variables.

70. $x^2 + (a + b)x + ab = (a + b)\ (x + ab)$

71. Common factor of $11pq^2$, $121p^2q^3$, $1331p^2q$ is $11p^2q^2$.

72. Common factor of $12a^2b^2 + 4ab^2 - 32$ is 4.

73. Factorisation of $-\ 3a^2 + 3ab + 3ac$ is $3a\ (-a - b - c)$.

74. Factorised form of $p^2 + 30p + 216$ is $(p + 18)\ (p - 12)$.

75. The difference of the squares of two consecutive numbers is their sum.

76. $abc + bca + cab$ is a monomial.

77. On dividing $\frac{p}{3}$ by $\frac{3}{p}$, the quotient is 9.

78. The value of p for $51^2 - 49^2 = 100p$ is 2.

79. $(9x - 51) \div 9$ is $x - 51$.

80. The value of $(a + 1)\ (a - 1)\ (a^2 + 1)$ is $a^4 - 1$.

81. Add:

(i) $7a^2bc, -3abc^2, 3a^2bc, 2abc^2$

(ii) $9ax, +3by - cz, -5by + ax + 3cz$

(iii) $xy^2z^2 + 3x^2y^2z - 4x^2yz^2, -9x^2y^2z + 3xy^2z^2 + x^2yz^2$

(iv) $5x^2 - 3xy + 4y^2 - 9, 7y^2 + 5xy - 2x^2 + 13$

(v) $2p^4 - 3p^3 + p^2 - 5p + 7, -3p^4 - 7p^3 - 3p^2 - p - 12$

(vi) $3a(a - b + c)$, $2b(a - b + c)$

(vii) $3a(2b + 5c)$, $3c(2a + 2b)$

82. Subtract :

(i) $5a^2b^2c^2$ from $-7a^2b^2c^2$

(ii) $6x^2 - 4xy + 5y^2$ from $8y^2 + 6xy - 3x^2$

(iii) $2ab^2c^2 + 4a^2b^2c - 5a^2bc^2$ from $-10a^2b^2c + 4ab^2c^2 + 2a^2bc^2$

(iv) $3t^4 - 4t^3 + 2t^2 - 6t + 6$ from $-4t^4 + 8t^3 - 4t^2 - 2t + 11$

(v) $2ab + 5bc - 7ac$ from $5ab - 2bc - 2ac + 10abc$

(vi) $7p(3q + 7p)$ from $8p(2p - 7q)$

(vii) $-3p^2 + 3pq + 3px$ from $3p(-p - a - r)$

83. Multiply the following:

(i) $-7pq^2r^3, -13p^3q^2r$

(ii) $3x^2y^2z^2, 17xyz$

(iii) $15xy^2, 17yz^2$

(iv) $-5a^2bc, 11ab, 13abc^2$

(v) $-3x^2y, (5y - xy)$

(vi) $abc, (bc + ca)$

(vii) $7pqr, (p - q + r)$

(viii) $x^2y^2z^2, (xy - yz + zx)$

(ix) $(p + 6), (q - 7)$

(x) $6mn, 0mn$

(xi) a, a^5, a^6

(xii) $-7st, -1, -13st^2$

(xiii) $b^3, 3b^2, 7ab^5$

(xiv) $-\frac{100}{9}rs; \frac{3}{4}r^3s^2$

(xv) $(a^2 - b^2), (a^2 + b^2)$

(xvi) $(ab + c), (ab + c)$

(xvii) $(pq - 2r), (pq - 2r)$

(xviii) $\left(\frac{3}{4}x - \frac{4}{3}y\right), \left(\frac{2}{3}x + \frac{3}{2}y\right)$

(xix) $\frac{3}{2}p^2 + \frac{2}{3}q^2$, $(2p^2 - 3q^2)$

(xx) $(x^2 - 5x + 6), (2x + 7)$

(xxi) $(3x^2 + 4x - 8), (2x^2 - 4x + 3)$

(xxii) $(2x - 2y - 3), (x + y + 5)$

84. Simplify

(i) $(3x + 2y)^2 + (3x - 2y)^2$

(ii) $(3x + 2y)^2 - (3x - 2y)^2$

(iii) $\left(\frac{7}{9}a + \frac{9}{7}b\right)^2 - ab$

(iv) $\left(\frac{3}{4}x - \frac{4}{3}y\right)^2 + 2xy$

(v) $(1.5p + 1.2q)^2 - (1.5p - 1.2q)^2$

(vi) $(2.5m + 1.5q)^2 + (2.5m - 1.5q)^2$

(vii) $(x^2 - 4) + (x^2 + 4) + 16$

(viii) $(ab - c)^2 + 2abc$

(ix) $(a - b)(a^2 + b^2 + ab) - (a + b)(a^2 + b^2 - ab)$

(x) $(b^2 - 49)(b + 7) + 343$

(xi) $(4.5a + 1.5b)^2 + (4.5b + 1.5a)^2$

(xii) $(pq - qr)^2 + 4pq^2r$

(xiii) $(s^2t + tq^2)^2 - (2stq)^2$

85. Expand the following, using suitable identities.

(i) $(xy + yz)^2$

(ii) $(x^2y - xy^2)^2$

(iii) $\left(\frac{4}{5}a + \frac{5}{4}b\right)^2$

(iv) $\left(\frac{2}{3}x - \frac{3}{2}y\right)^2$

(v) $\left(\frac{4}{5}p + \frac{5}{3}q\right)^2$

(vi) $(x + 3)(x + 7)$

(vii) $(2x + 9)(2x - 7)$

(viii) $\left(\frac{4x}{5} + \frac{y}{4}\right)\left(\frac{4x}{5} + \frac{3y}{4}\right)$

(ix) $\left(\frac{2x}{3} - \frac{2}{3}\right)\left(\frac{2x}{3} + \frac{2a}{3}\right)$

(x) $(2x - 5y)(2x - 5y)$

(xi) $\left(\frac{2a}{3} + \frac{b}{3}\right)\left(\frac{2a}{3} - \frac{b}{3}\right)$

(xii) $(x^2 + y^2)(x^2 - y^2)$

(xiii) $(a^2 + b^2)^2$

(xiv) $(7x + 5)^2$

(xv) $(0.9p - 0.5q)^2$

(xvi) $x^2y^2 = (xy)^2$

86. Using suitable identities, evaluate the following.

(i) $(52)^2$ (ii) $(49)^2$

(iii) $(103)^2$ (iv) $(98)^2$

(v) $(1005)^2$ (vi) $(995)^2$

(vii) 47×53 (viii) 52×53

(ix) 105×95 (x) 104×97

(xi) 101×103 (xii) 98×103

(xiii) $(9.9)^2$ (xiv) 9.8×10.2

(xv) 10.1×10.2 (xvi) $(35.4)^2 - (14.6)^2$

(xvii) $(69.3)^2 - (30.7)^2$ (xviii) $(9.7)^2 - (0.3)^2$

(xix) $(132)^2 - (68)^2$ (xx) $(339)^2 - (161)^2$

(xxi) $(729)^2 - (271)^2$

87. Write the greatest common factor in each of the following terms.

(i) $-18a^2$, $108a$ (ii) $3x^2y$, $18xy^2$, $-6xy$

(iii) $2xy$, $-y^2$, $2x^2y$ (iv) l^2m^2n, lm^2n^2, l^2mn^2

(v) $21pqr$, $-7p^2q^2r^2$, $49p^2qr$ (vi) $qrxy$, $pryz$, $rxyz$

(vii) $3x^3y^2z$, $-6xy^3z^2$, $12x^2yz^3$

(viii) $63p^2a^2r^2s$, $-9pq^2r^2s^2$, $15p^2qr^2s^2$, $-60p^2a^2rs^2$

(ix) $13x^2y$, $169xy$

(x) $11x^2$, $12y^2$

88. Factorise the following expressions.

(i) $6ab + 12bc$ (ii) $-xy - ay$

(iii) $ax^3 - bx^2 + cx$ (iv) $l^2m^2n - lm^2n^2 - l^2mn^2$

(v) $3pqr - 6p^2q^2r^2 - 15r^2$ (vi) $x^3y^2 + x^2y^3 - xy^4 + xy$

(vii) $4xy^2 - 10x^2y + 16x^2y^2 + 2xy$

(viii) $2a^3 - 3a^2b + 5ab^2 - ab$

(ix) $63p^2q^2r^2s - 9pq^2r^2s^2 + 15p^2qr^2s^2 - 60p^2q^2rs^2$

(x) $24x^2yz^3 - 6xy^3z^2 + 15x^2y^2z - 5xyz$

(xi) $a^3 + a^2 + a + 1$

(xii) $lx + my + mx + ly$

(xiii) $a^3x - x^4 + a^2x^2 - ax^3$

(xiv) $2x^2 - 2y + 4xy - x$

(xv) $y^2 + 8zx - 2xy - 4yz$

(xvi) $ax^2y - bxyz - ax^2z + bxy^2$

(xvii) $a^2b + a^2c + ab + ac + b^2c + c^2b$

(xviii) $2ax^2 + 4axy + 3bx^2 + 2ay^2 + 6bxy + 3by^2$

89. Factorise the following, using the identity $a^2 + 2ab + b^2 = (a + b)^2$

(i) $x^2 + 6x + 9$

(ii) $x^2 + 12x + 36$

(iii) $x^2 + 14x + 49$

(iv) $x^2 + 2x + 1$

(v) $4x^2 + 4x + 1$

(vi) $a^2x^2 + 2ax + 1$

(vii) $a^2x^2 + 2abx + b^2$

(viii) $a^2x^2 + 2abxy + b^2y^2$

(ix) $4x^2 + 12x + 9$

(x) $16x^2 + 40x + 25$

(xi) $9x^2 + 24x + 16$

(xii) $9x^2 + 30x + 25$

(xiii) $2x^3 + 24x^2 + 72x$

(xiv) $a^2x^3 + 2abx^2 + b^2x$

(xv) $4x^4 + 12x^3 + 9x^2$

(xvi) $\frac{x^2}{4} + 2x + 4$

(xvii) $9x^2 + 2xy + \frac{y^2}{9}$

90. Factorise the following, using the identity $a^2 - 2ab + b^2 = (a - b)^2$.

(i) $x^2 - 8x + 16$

(ii) $x^2 - 10x + 25$

(iii) $y^2 - 14y + 49$

(iv) $p^2 - 2p + 1$

(v) $4a^2 - 4ab + b^2$

(vi) $p^2y^2 - 2py + 1$

(vii) $a^2y^2 - 2aby + b^2$

(viii) $9x^2 - 12x + 4$

(ix) $4y^2 - 12y + 9$

(x) $\frac{x^2}{4} - 2x + 4$

(xi) $a^2y^3 - 2aby^2 + b^2y$ (xii) $9y^2 - 4xy + \frac{4x^2}{9}$

91. Factorise the following.

(i) $x^2 + 15x + 26$ (ii) $x^2 + 9x + 20$

(iii) $y^2 + 18x + 65$ (iv) $p^2 + 14p + 13$

(v) $y^2 + 4y - 21$ (vi) $y^2 - 2y - 15$

(vii) $18 + 11x + x^2$ (viii) $x^2 - 10x + 21$

(ix) $x^2 = 17x + 60$ (x) $x^2 + 4x - 77$

(xi) $y^2 + 7y + 12$ (xii) $p^2 - 13p - 30$

(xiii) $a^2 - 16p - 80$

92. Factorise the following using the identity $a^2 - b^2 = (a + b)(a - b)$.

(i) $x^2 - 9$ (ii) $4x^2 - 25y^2$

(iii) $4x^2 - 49y^2$ (iv) $3a^2b^3 - 27a^4b$

(v) $28ay^2 - 175ax^2$ (vi) $9x^2 - 1$

(vii) $25ax^2 - 25a$ (viii) $\frac{x^2}{9} - \frac{y^2}{25}$

(ix) $\frac{2p^2}{25} - 32q^2$ (x) $49x^2 - 36y^2$

(xi) $y^3 - \frac{y}{9}$ (xii) $\frac{x^2}{25} - 625$

(xiii) $\frac{x^2}{8} - \frac{y^2}{18}$ (xiv) $\frac{4x^2}{9} - \frac{9y^2}{16}$

(xv) $\frac{x^3y}{9} - \frac{xy^3}{16}$ (xvi) $1331x^3y - 11y^3x$

(xvii) $\frac{1}{36}a^2b^2 - \frac{16}{49}b^2c^2$ (xviii) $a^4 - (a - b)^4$

(xix) $x^4 - 1$ (xx) $y^4 - 625$

(xxi) $p^5 - 16p$

(xxii) $16x^4 - 81$

(xxiii) $x^4 - y^4$

(xxiv) $y^4 - 81$

(xxv) $16x^4 - 625y^4$

(xxvi) $(a - b)^2 - (b - c)^2$

(xxvii) $(x + y)^4 - (x - y)^4$

(xxviii) $x^4 - y^4 + x^2 - y^2$

(xxix) $8a^3 - 2a$

(xxx) $x^2 - \dfrac{y^2}{100}$

(xxxi) $9x^2 - (3y + z)^2$

93. The following expressions are the areas of rectangles. Find the possible lengths and breadths of these rectangles.

(i) $x^2 - 6x + 8$

(ii) $x^2 - 3x + 2$

(iii) $x^2 - 7x + 10$

(iv) $x^2 + 19x - 20$

(v) $x^2 + 9x + 20$

94. Carry out the following divisions:

(i) $51x^3y^2z \div 17xyz$

(ii) $76x^3yz^3 \div 19x^2y^2$

(iii) $17ab^2c^3 \div (-abc^2)$

(iv) $-121p^3q^3r^3 \div (-11xy^2z^3)$

95. Perform the following divisions:

(i) $(3pqr - 6p^2q^2r^2) \div 3pq$

(ii) $(ax^3 - bx^2 + cx) \div (-dx)$

(iii) $(x^3y^3 + x^2y^3 - xy^4 + xy) \div xy$

(iv) $(-qrxy + pryz - rxyz) \div (-xyz)$

96. Factorise the expressions and divide them as directed:

(i) $(x^2 - 22x + 117) \div (x - 13)$

(ii) $(x^3 + x^2 - 132x) \div x(x - 11)$

(iii) $(2x^3 - 12x^2 + 16x) \div (x - 2)(x - 4)$

(iv) $(9x^2 - 4) \div (3x + 2)$

(v) $(3x^2 - 48) \div (x - 4)$

(vi) $(x^4 - 16) \div x^3 + 2x^2 + 4x + 8$

(vii) $(3x^4 - 1875) \div (3x^2 - 75)$

97. The area of a square is given by $4x^2 + 12xy + 9y^2$. Find the side of the square.

98. The area of a square is $9x^2 + 24xy + 16y^2$. Find the side of the square.

99. The area of a rectangle is $x^2 + 7x + 12$. If its breadth is $(x + 3)$, then find its length.

100. The curved surface area of a cylinder is $2\pi (y^2 - 7y + 12)$ and its radius is $(y - 3)$. Find the height of the cylinder (C.S.A. of cylinder = $2\pi rh$).

101. The area of a circle is given by the expression $\pi x^2 + 6\pi x + 9\pi$. Find the radius of the circle.

102. The sum of first n natural numbers is given by the expression $\frac{n^2}{2} + \frac{n}{2}$. Factorise this expression.

103. The sum of $(x + 5)$ observations is $x^4 - 625$. Find the mean of the observations.

104. The height of a triangle is $x^4 + y^4$ and its base is $14xy$. Find the area of the triangle.

105. The cost of a chocolate is Rs $(x + y)$ and Rohit bought $(x + y)$ chocolates. Find the total amount paid by him in terms of x. If x = 10, find the amount paid by him.

106. The base of a parallelogram is $(2x + 3$ units) and the corresponding height is $(2x - 3$ units). Find the area of the parallelogram in terms of x. What will be the area of parallelogram of $x = 30$ units?

107. The radius of a circle is $7ab - 7bc - 14ac$. Find the circumference of the circle. $\left(\pi = \frac{22}{7}\right)$

108. If $p + q = 12$ and $pq = 22$, then find $p^2 + q^2$.

109. If $a + b = 25$ and $a^2 + b^2 = 225$, then find ab.

110. If $x - y = 13$ and $xy = 28$, then find $x^2 + y^2$.

111. If $m - n = 16$ and $m^2 + n^2 = 400$, then find mn.

112. If $a^2 + b^2 = 74$ and $ab = 35$, then find $a + b$.

113. Verify the following:

(i) $(ab + bc)(ab - bc) + (bc + ca)(bc - ca) + (ca + ab)(ca - ab) = 0$

(ii) $(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = a^3 + b^3 + c^3 - 3abc$

(iii) $(p - q)(p^2 + pq + q^2) = p^3 - q^3$

(iv) $(m + n)(m^2 - mn + n^2) = m^3 + n^3$

(v) $(a + b)(a + b)(a + b) = a^3 + 3a^2b + 3ab^2 + b^3$

(vi) $(a - b)(a - b)(a - b) = a^3 - 3a^2b + 3ab^2 - b^3$

(vii) $(a^2 - b^2)(a^2 + b^2) + (b^2 - c^2)(b^2 + c^2) + (c^2 - a^2) + (c^2 + a^2) = 0$

(viii) $(5x + 8)^2 - 160x = (5x - 8)^2$

(ix) $(7p - 13q)^2 + 364pq = (7p + 13q)^2$

(x) $\left(\frac{3p}{7}+\frac{7}{6p}\right)^2-\left(\frac{3}{7}p+\frac{7}{6p}\right)^2 = 2$

114. Find the value of a, *if*

(i) $8a = 35^2 - 27^2$

(ii) $9a = 76^2 - 67^2$

(iii) $pqa = (3p + q)^2 - (3p - q)^2$

(iv) $pq^2a = (4pq + 3q)^2 - (4pq - 3q)^2$

115. What should be added to $4c(-a + b + c)$ to obtain $3a(a + b + c) - 2b(a - b + c)$?

116. Subtract $b(b^2 + b - 7) + 5$ from $3b^2 - 8$ and find the value of expression obtained for $b = -3$.

117. If $x-\frac{1}{x}=7$ then find the value of $x^2+\frac{1}{x^2}$.

118. Factorise $x^2+\frac{1}{x^2}+2-3x-\frac{3}{x}$.

119. Factorise $p^4 + q^4 + p^2q^2$.

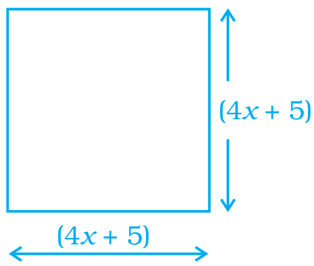
120. Find the value of

(i) $\frac{6.25\times6.25-1.75\times1.75}{4.5}$

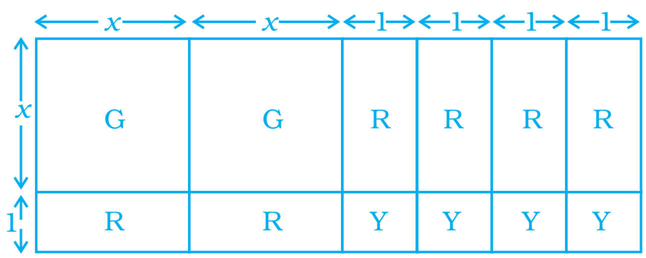
(ii) $\frac{198\times198-102\times102}{96}$

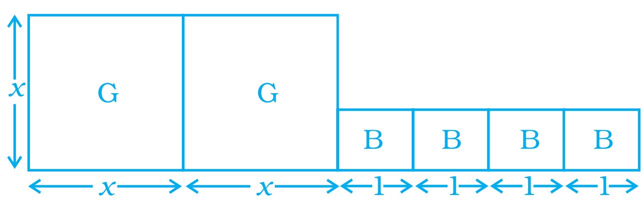
121. The product of two expressions is $x^5 + x^3 + x$. If one of them is $x^2 + x + 1$, find the other.

122. Find the length of the side of the given square if area of the square is 625 square units and then find the value of x.



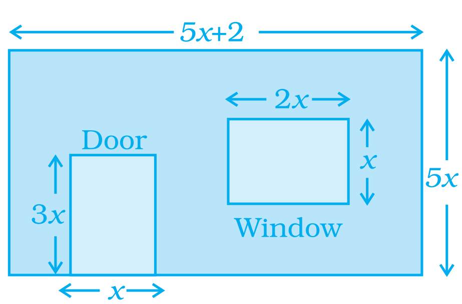
123. Take suitable number of cards given in the adjoining diagram [G($x \times x$) representing x^2, R ($x \times 1$) representing x and Y (1×1) representing 1] to factorise the following expressions, by arranging the cards in the form of rectangles: (i) $2x^2 + 6x + 4$ (ii) $x^2 + 4x + 4$. Factorise $2x^2 + 6x + 4$ by using the figure.





Calculate the area of figure.

124. The figure shows the dimensions of a wall having a window and a door of a room. Write an algebraic expression for the area of the wall to be painted.



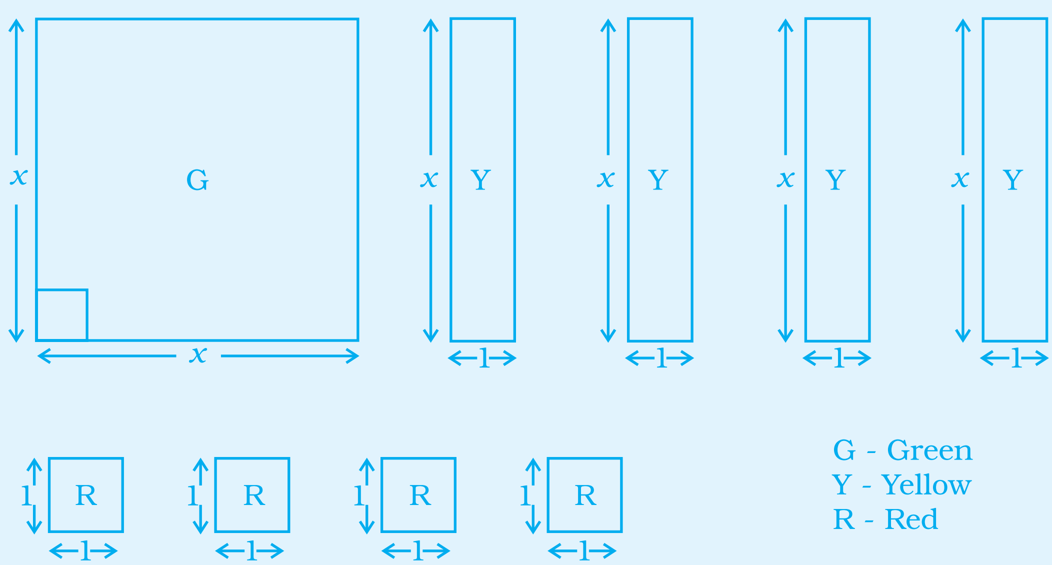
125. Match the expressions of column I with that of column II:

Column I	**Column II**
(1) $(21x + 13y)^2$	(a) $441x^2 - 169y^2$
(2) $(21x - 13y)^2$	(b) $441x^2 + 169y^2 + 546\,xy$
(3) $(21x - 13y)\,(21x + 13y)$	(c) $441x^2 + 169y^2 - 546xy$
	(d) $441x^2 - 169y^2 + 546xy$

(D) ACTIVITIES

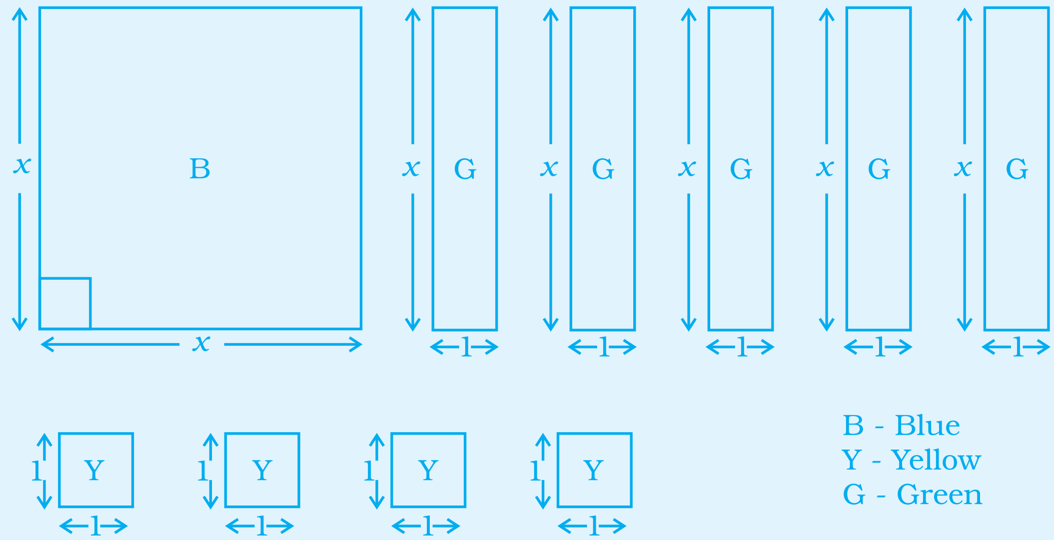
1. Algebraic Tiles

(i) Cut the following tiles from a graph sheet. Now, colour the tiles as per the colour code. Arrange these algebraic tiles to form a square.



Find the length of the side of the square so formed. Also find the area of the square. Using the above result factorise $x^2 + 4x + 4$.

(ii)



2. Find the length of the side of the rectangle so formed. Also find the area of the rectangle. Using the above result factorise $x^2 + 5x + 4$.

   Now choose and cut more algebraic tiles from the graph sheet. Create your own colour code and colour the tiles. Arrange them to form square/rectangle. Find the area of the figure so formed using it to factorise

   a) $x^2 + 4x + 3$

   b) $x^2 + 9x + 18$

3. Build a square garden. Divide the square garden into four rectangular flower beds in such a way that each flower bed is as long as one side of the square. The perimeter of each flower bed is 40 m.

   (a) Draw a diagram to represent the above information.

   (b) Mention the expression for perimeter of the entire garden.

Crossword Number Puzzle

Solve the given crossword and then fill up the given boxes. Clues are given below for across as well as downward filling. Also for across and down clues clue number is written at the corner of boxes. Answers of clues have to be filled in their respective boxes.

Down

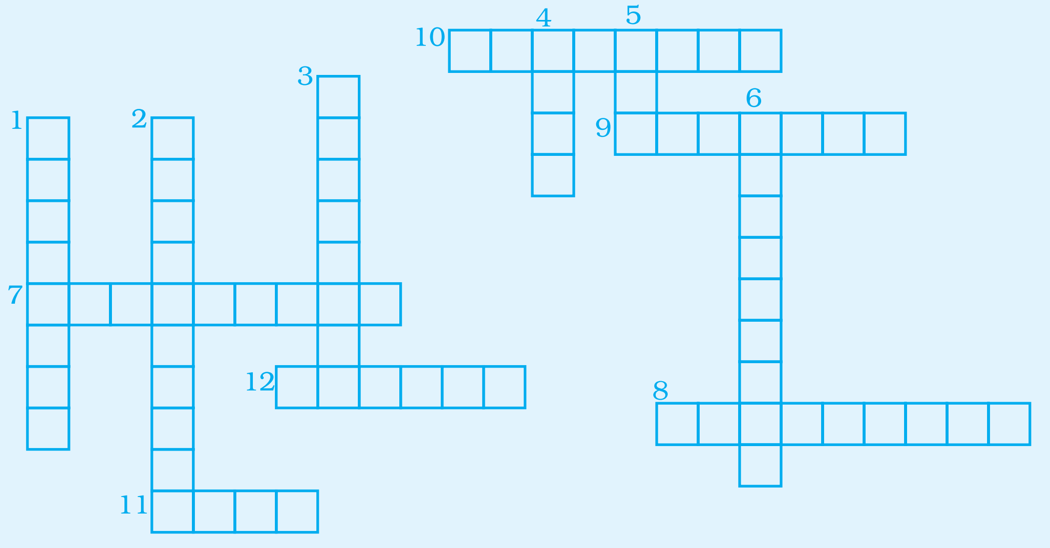
1. A polynomial with two terms.
2. An expression containing one or more terms with non-zero coefficient (with variables having non-negative exponents).
3. To find the value of a mathematical expression.
4. A _________ is formed by the product of variables and constants.
5. The abbreviation of the greatest no. (or expression) that in a factor of two or more numbers.
6. A polynomial with three terms.

Across

7. A polynomial with only one term.
8. An expression of the second degree.
9. Terms can be written as product of its _________.
10. The numbers –3, –2, –1, 0, 1, 2, 3 are known as _______.
11. __________ terms are formed from the same variables and the powers of these variables are the same term.
12. The highest power of a polynomial is called the _________ of the polynomial.

Solution

1. Binomial
2. Polynomial
3. Evaluate
4. Term
5. GCF
6. Trinomial
7. Monomial



8. Quadratic
9. Factors
10. Integers
11. Like
12. Degree

Rough Work