UNIT 3

SQUARE-SQUARE ROOT AND CUBE-CUBE ROOT

(A) Main Concepts and Results

- A natural number is called a ***perfect square*** if it is the square of some natural number.

  i.e., if $m = n^2$, then m is a perfect square where m and n are natural numbers.

- A natural number is called a ***perfect cube*** if it is the cube of some natural number.

  i.e., if $m = n^3$, then m is a perfect cube where m and n are natural numbers.

- Number obtained when a number is multiplied by itself is called the square of the number.
- Number obtained when a number is multiplied by itself three times are called ***cube number***.
- Squares and cubes of even numbers are even.
- Squares and cubes of odd numbers are odd.
- A perfect square can always be expressed as the product of pairs of prime factors.
- A perfect cube can always be expressed as the product of triplets of prime factors.

- The unit digit of a perfect square can be only 0, 1, 4, 5, 6 or 9.
- The square of a number having:

  1 or 9 at the units place ends in1.

  2 or 8 at the units place ends in 4.

  3 or 7 at the units place ends in 9.

  4 or 6 at the units place ends in 6.

  5 at the units place ends in 5.
- There are $2n$ natural numbers between the squares of numbers n and $n+1$.
- A number ending in odd numbers of zeroes is not a perfect square.
- The sum of first n odd natural numbers is given by n^2.
- Three natural numbers a, b, c are said to form a pythagorean triplet if $a^2 + b^2 = c^2$.
- For every natural number $m > 1$, $2m$, m^2-1 and $m^2 + 1$ form a pythagorean triplet.
- The square root of a number x is the number whose square is x. Positive square root of a number x is denoted by $\sqrt{x}$.
- The cube root of a number x is the number whose cube is x. It is denoted by $\sqrt[3]{x}$.
- Square root and cube root are the inverse operations of squares and cubes respectively.
- If a perfect square is of n digits, then its square root will have $\frac{n}{2}$ digit if n is even or $\left(\frac{n+1}{2}\right)$ digit if n is odd.
- Cubes of the numbers ending with the digits 0, 1, 4, 5, 6 and 9 end with digits 0, 1, 4, 5, 6 and 9 respectively.

Think and Discuss

1. **Describe** what is meant by a perfect square. Give an example.
2. **Explain** how many square roots a positive number can have. How are these square roots different?

Key Concept *To be Noted*

SQUARE ROOTS

Words	A square root of a number n is a number m which, when multiplied by itself, equals n.
Numbers	The square roots of 16 are 4 and –4 because $4^2 = 16$ and $(-4)^2 = 16$.
Algebra	If $m^2 = n$, then m is a square root of n.

Think and Discuss

1. Which type of number has an exact square root?
2. Which type of number has an approximate square root?
3. How can we use perfect squares to estimate a square root, such as $\sqrt{8}$?

- Cube of the number ending in 2 ends in 8 and cube root of the number ending in 8 ends in 2.
- Cube of the number ending in 3 ends in 7 and cube root of the number ending in 7 ends in 3.

(B) Solved Examples

In examples 1 to 7, out of given four choices only one is correct. Write the correct answer.

Example 1 : Which of the following is the square of an odd number?

(a) 256 (b) 361 (c) 144 (d) 400

Solution : Correct answer is (b).

Example 2 : Which of the following will have 1 at its units place?

(a) 19^2 (b) 17^2 (c) 18^2 (d) 16^2

Solution : Correct answer is (a).

Example 3 : How many natural numbers lie between 18^2 and 19^2?

(a) 30 (b) 37 (c) 35 (d) 36

Solution : Correct answer is (d).

Example 4 : Which of the following is not a perfect square?

(a) 361 (b) 1156 (c) 1128 (d) 1681

Solution : Correct answer is (c).

Example 5 : A perfect square can never have the following digit at ones place.

(a) 1 (b) 6 (c) 5 (d) 3

Solution : Correct answer is (d).

Example 6 : The value of $\sqrt{176+\sqrt{2401}}$ is

(a) 14 (b) 15 (c) 16 (d) 17

Solution : Correct answer is (b).

$$\left(\sqrt{176+\sqrt{2401}} = \sqrt{176+49} = \sqrt{225} = 15\right)$$

Example 7 : Given that $\sqrt{5625}$ =75, the value of $\sqrt{0.5625}+\sqrt{56.25}$ is:

(a) 82.5 (b) 0.75 (c) 8.25 (d) 75.05

Solution : Correct answer is (c).

If ($\sqrt{5625}$ = 75, then $\sqrt{0.5625}$ = 0.75 and $\sqrt{56.25}$ = 7.5)

In examples 8 to 14, fill in the blanks to make the statements true.

Example 8 : There are __________ perfect squares between 1 and 50.

Solution : 6

Example 9 : The cube of 100 will have __________ zeroes.

Solution : 6

Example 10 : The square of 6.1 is ____________.

Solution : 37.21

1. Squaring a number and taking a square root are inverse operations. What other inverse operations do you know?
2. When the factors of a perfect square are written in order from the least to greatest, what do you notice?
3. Why do you think numbers such as 4, 9, 16, ... are called perfect squares?
4. Suppose you list the factors of a perfect square. Why is one factor square root and not the other factors?

Example 11 : The cube of 0.3 is ____________.

Solution : 0.027

Connect

here are some ways to tell whether a number is a square number.

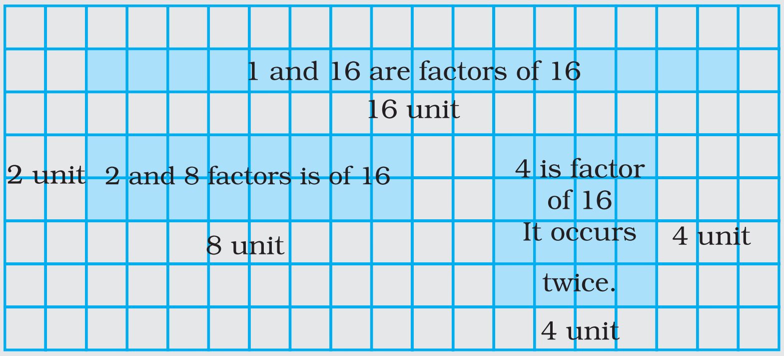
- If we can find a division sentence for a number so that the quotient is equal to the divisor, the number is a square number.

  For example, $\underset{\substack{\uparrow \\ \text{dividend}}}{16} \div \underset{\substack{\uparrow \\ \text{divisor}}}{4} = \underset{\substack{\nwarrow \\ \text{quotient}}}{4}$, so 16 is a square number.

- We can also use factoring.

  Factors of a number occur in pairs.

  These are the dimensions of a rectangle.



Sixteen has 5 factors: 1, 2, 4, 8, 16

Since there is an odd number of factors, one rectangle is a square.

A factor that occurs twice is only written once in the list of factors.

The square has side length of 4 units.

We say that 4 is a square root of 16.

We write $4 = \sqrt{16}$

When a number has an odd number of factors, it is a square number.

Think and Discuss

1. **Discuss** whether 9.5 is a good first guess for $\sqrt{75}$.
2. **Determine** which square root or roots would have 7.5 as a good first guess.

Example 12 : 68^2 will have __________ at the units place.

Solution : 4

Example 13 : The positive square root of a number x is denoted by __________.

Solution : $\sqrt{x}$

Example 14 : The least number to be multiplied with 9 to make it a perfect cube is _______________.

Solution : 3

In examples 15 to 19, state whether the statements are true (T) or false (F)

Example 15 : The square of 0.4 is 0.16.

Solution : True

Example 16 : The cube root of 729 is 8.

Solution : False

Example 17 : There are 21 natural numbers between 10^2 and 11^2.

Solution: False

Example 18 : The sum of first 7 odd natural numbers is 49.

Solution : True

Example 19 : The square root of a perfect square of n digits will have $\frac{n}{2}$ digits if n is even.

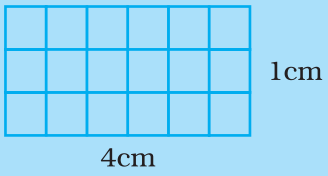
Solution : True

Example 20 : Express 36 as a sum of successive odd natural numbers.

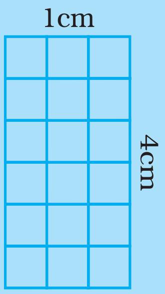
Solution : $1+3+5+7+9+11 = 36$

A rectangle is a quadrilateral with 4 right angles.
A square also has 4 right angles.

A rectangle with base 4 cm and height 1cm is the same as a rectangle with base 1cm and height 4 cm.

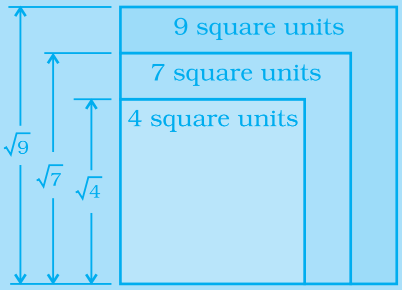


These two rectangles are congruent.
Is every square a rectangle?
Is every rectangle a square?



Copy this diagram on grid paper.

Then estimate the value of $\sqrt{7}$ to one decimal place.



Example 21 : Check whether 90 is a perfect square or not by using prime factorisation.

Solution : Prime factorisation of 90 is

2	90
3	45
3	15
5	5
	1

$90 = 2 \times 3 \times 3 \times 5$

The prime factors 2 and 5 do not occur in pairs. Therefore, 90 is not a perfect square.

Example 22 : Check whether 1728 is a perfect cube by using prime factorisation.

Solution : Prime factorisation of 1728 is

$1728 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$

Since all prime factors can be grouped in triplets. Therefore, 1728 is a perfect cube.

Apply

Use square tiles. Make as many different rectangles as you can with area 28 square units.
Draw your rectangles on grid paper.
Is 28 a perfect square? Justify your answer.

Example 23 : Using distributive law, find the square of 43.

Solution : $43 = 40 + 3$

So $43^2 = (40 + 3)^2 = (40 + 3)\ (40 + 3) = 40\ (40 + 3) + 3(40 + 3)$

$= 40 \times 40 + 40 \times 3 + 3 \times 40 + 3 \times 3$

$= 1600 + 240 + 9$

$= 1849$

So, $43^2 = 1849$

Example 24 : Write a pythagorean triplet whose smallest number is 6.

Solution : Smallest number is 6

$2m = 6$ or $m = 3$

$m^2 + 1 = 3^2 + 1 = 9 + 1 = 10$

$m^2 - 1 = 3^2 - 1 = 9 - 1 = 8$

So, the pythagorean triplet is 6, 8, 10.

Connect

Here is one way to estimate the value of $\sqrt{20}$:

- 25 is the square number closest to 20, but greater than 20.

  On grid paper, draw a square with area 25.

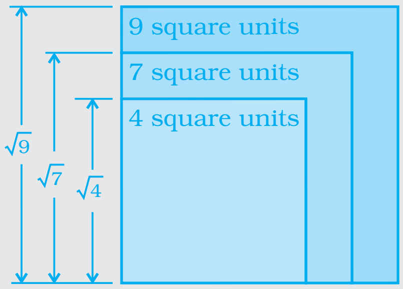
  Its side length: $\sqrt{25} = 5$

- 16 is the square number closest to 20, but less than 20.

  Draw a square with area 16

  Its side length: $\sqrt{16} = 4$

Draw the squares so that they overlap.



A square with area 20 lies between these two squares.

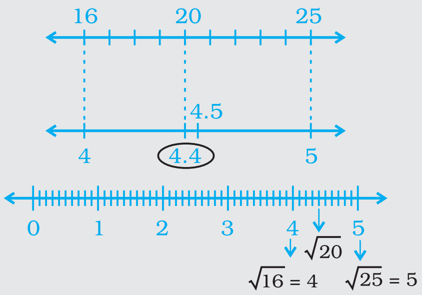
Its side length $\sqrt{20}$.

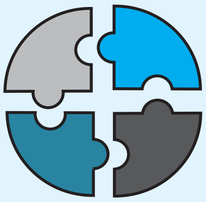
20 is between 16 and 25, but closer to 16.

So, $\sqrt{20}$ is between $\sqrt{16}$ and $\sqrt{25}$, but closer to $\sqrt{16}$.

So, $\sqrt{20}$ is between 4 and 5, but closer to 4.

An estimate of $\sqrt{20}$ is 4.4 to one decimal place.





Application on Problem Solving Strategy

A couple wants to install a square glass window that has an area of 500 square cm. Calculate the length of each side and the length of trim needed to the nearest tenth of cm.

Understand the problem

First find the length of a side. Then you can use the length of the side to find the perimeter – the length of the trim around the window.

Make a Plan

The length of a side, in cm, is the number that you multiply by itself to get 500. Find this number to the nearest tenth.

Use guess and check to find $\sqrt{500}$.

Solve

Because 5000 is between 22^2 (484) and 23^2 (529), the square root of 500 is between 22 and 23.

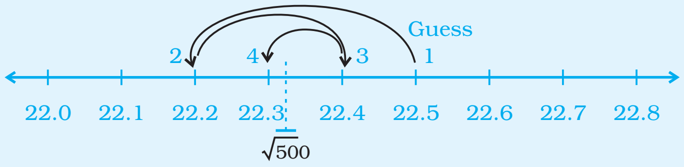
The square root is between 22.3 and 22.4. To round to the nearest tenth, consider 22.35.

$22.35^2 = 499.5225$ *low*

Guess 22.5	**Guess 22.2**	**Guess 22.4**	**Guess 22.3**
22.5^2 506.25	22.2^2 492.84	22.4^2 501.76	22.3^2 497.29
high	low	high	low
Square root is between 22 and 22.5	Square root is between 22.2 and 22.5	Square root is between 22.2 and 22.4	Square root is between 22.3 and 22.4

The square root must be greater than 22.35, so you can round up.

To the nearest tenth, $\sqrt{500}$ is about 22.4.



Now estimate the length around the window. The length of a side of the window to the nearest tenth of an inch is 22.4 inches.

$4 \times 22.4 = 89.6$ *(Perimeter = 4 × side)*

The trim is about 89.6 cm long.

Look Back

The length 90 cm divided by 4 is 22.5 cm. A 22.5 cm square has an area of 506 square cm, which is close to 500, so the answers are reasonable.

Example 25 : Using prime factorisation, find the cube root of 5832.

Solution : The prime factorisation of 5832 is

2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

$$5832 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$$

Therefore, $\sqrt[3]{5832} = \sqrt[3]{2\times2\times2\times3\times3\times3\times3\times3\times3}$

$= 2 \times 3 \times 3$

$= 18$

Take It Further

a) Find the square root of each palindromic number.
A palindromic number is a number that reads the same – forward and backward.

(i) $\sqrt{121}$

(ii) $\sqrt{12\,321}$

(iii) $\sqrt{1\,234\,321}$

(iv) $\sqrt{123\,454\,321}$

b) Continue the pattern.
Write the next 4 palindromic numbers in the pattern and their square roots.

Think and Discuss

1. Is 1 a square number? How can you tell?
2. Suppose you know the area of a square. How can you find its perimeter?
3. Suppose you know the perimeter of a square. How can you find its area?

Example 26 : Evaluate the square root of 22.09 by long division method.

Solution : 4.7

$$\begin{array}{r|l} 4 & 22.09 \\ & 16 \\ \hline 87 & 609 \\ & 609 \\ \hline & 0 \end{array}$$

Therefore, $\sqrt{22.09} = 4.7$

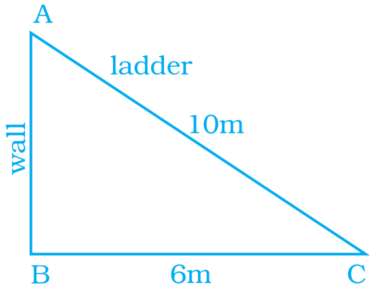
Example 27 : Find the smallest perfect square divisible by 3, 4, 5 and 6.

Solution : The least number divisible by 3, 4, 5 and 6 is their LCM. The LCM of 3, 4, 5 and 6 is 60. Now, $60 = 2 \times 2 \times 5 \times 3$.

We see that prime factors 5 and 3 are not in pairs. Therefore 60 is not a perfect square. So, 60 should be multiplied by $5 \times 3 = 15$ to get a perfect square.

Thus, the required least square number $= 60 \times 15 = 900$.

Example 28 : A ladder 10m long rests against a vertical wall. If the foot of the ladder is 6m away from the wall and the ladder just reaches the top of the wall, how high is the wall?



Solution : Let AC be the ladder.

Therefore, AC = 10m

Let BC be the distance between the foot of the ladder and the wall.

Therefore,BC = 6m

ΔABC forms a right angled triangle, right angled at B.

By Pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$10^2 = AB^2 + 6^2$$

or $$AB^2 = 10^2 - 6^2 = 100 - 36 = 64$$

or $$AB = \sqrt{64} = 8\text{m}$$

Hence, the wall is 8m high.

Example 29 : Find the length of a diagonal of a rectangle with dimensions 20m by 15m.

Solution : Using Pythagoras theorem, we have

Length of diagonal of the rectangle

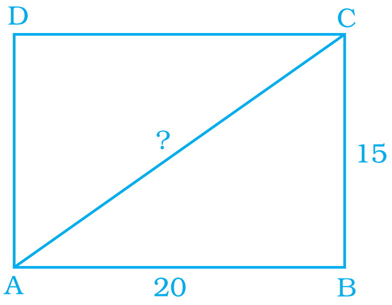
$= \sqrt{(l^2 + b^2)}$ units

$= \sqrt{(20^2 + 15^2)}$ m

$= \sqrt{400 + 225}$ m

$= \sqrt{625}$ m

= 25 m



Hence, the length of diagonal is 25m.

Investigate

Work with a partner.

You will need grid paper and 20 square tiles.

Use the tiles to make as many different rectangles as you can with each area.

4 square units
6 square units
8 square units
9 square units
12 square units
16 square units
20 square units

Draw the rectangles on grid paper.

- For how many areas given above were you able to make a square?
- What is the side length of each square you made?
- How is the side length of a square related to its area?

Think and Discuss

Compare your strategies and results with those of another pair of classmates.

Find two areas greater than 20 square units for which you could use tiles to make a square.

How do you know you could make a square for each of these areas?

Example 30 : The area of a rectangular field whose length is twice its breadth is 2450 m^2. Find the perimeter of the field.

Solution : Let the breadth of the field be x metres. Then length of the field is $2x$ metres.

Therefore, area of the rectangular field = length × breadth

$$= (2x)(x) = (2x^2)\ m^2$$

Given that area is 2450 m^2.

Therefore, $2x^2 = 2450$

$$x^2 = \frac{2450}{2}$$

$$x = \sqrt{1225} \text{ or } x = 35m$$

Hence, breadth = $35m$ and length $35 \times 2 = 70m$

Perimeter of the field = $2\ (l + b)$

$$= 2(70+35)m = 2\times105m = 210m$$

Example 31 : During a mass drill exercise, 6250 students of different schools are arranged in rows such that the number of students in each row is equal to the number of rows. In doing so, the instructor finds out that 9 children are left out. Find the number of children in each row of the square.

Solution : Total number of students = 6250

Number of students forming a square = 6250 – 9

= 6241

Thus, 6241 students form a big square which has number of rows equal to the number of students in each row.

Let the number of students in each row be x, then the number of rows = x

Therefore, $x \times x = 6241$

or $x = \sqrt{6241} = 79$

Hence, there are 79 students in each row of the square formed.

Example 32 : Find the least number that must be added to 1500 so as to get a perfect square. Also find the square root of the perfect square.

Solution :

$$\begin{array}{r|l} & 38 \\ \hline 3 & 1500 \\ & 9 \\ \hline 68 & 600 \\ & 544 \\ \hline & 56 \end{array}$$

We observe that $38^2 < 1500 < 39^2$

Hence the number to be added $= 39^2 - 1500$

$= 1521 - 1500$

$= 21$

Therefore, the perfect square is $1500 + 21 = 1521$

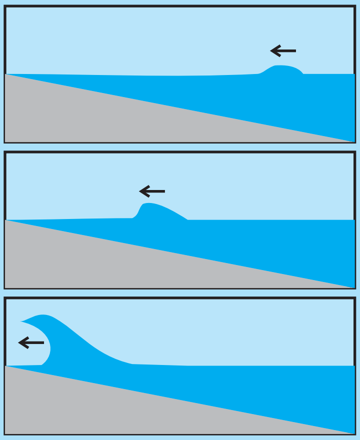
$\sqrt{1521} = 39$

Hence the required number is 21 and the square root is 39.

Tsunamis, sometimes called tidal waves, move across deep oceans at high speeds with barely a ripple on the water surface. It is only when tsunamis hit shallow water that their energy moves them upward into a huge destructive force.

Tsunamis can be caused by earthquakes, volcanoes, landslides, or meteorites.

1. The speed of a tsunami, in metre per second, can be found by the formula r = $\sqrt{9.7344d}$, where d is the water depth in metre. Suppose the water depth is 6400m. How fast is the tsunami moving?
2. The speed of a tsunami in km per hour can be found using $r = \sqrt{4.4944d}$ where d is the water depth in metre. Suppose the water depth is 8100 metre
   - a) How fast is the tsunami moving in km per hour?
   - b) How long would it take a tsunami to travel 3000 km if the water depth was a consistent 3000 m?



As the wave approaches the beach, it slows, builds in height and crashes on shore

Example 33 : Application of problem solving strategies

- Find the smallest number by which 1620 must be divided to get a perfect square.

Solution : Understand and Explore

- What information is given in the question? – A number which is not a perfect square.
- What are you trying to find? – The smallest number by which 1620 must be divided to get a perfect square.

Plan a strategy

- You have already learnt prime factorisation. Use it to find the product of prime factors of 1620.
- Pair the prime factors to see if any factor is left unpaired.
- This unpaired factor will be the smallest number that must be divided to get a perfect square.

Solve

Prime factorisation of 1620 is

2	1620
2	810
5	405
3	81
3	27
3	9
3	3
	1

The product of prime factors = 2 × 2 × 5 × 3 × 3 × 3 × 3

Pair these prime factors = **2 × 2** × 5 × **3 × 3** × **3 × 3**

The factor 5 is left unpaired.

Hence, the required smallest number is 5.

Revise

Divide 1620 by 5 and check if it is a perfect square.

1620 ÷ 5 = 324

We see that 324 is a perfect square, hence our answer is verified.

Think and Discuss

1. Find the square root of the number obtained in step IV.
2. Can you find the smallest number that can be multiplied to 1620 to get a perfect square?
3. Find the square root.

(C) Exercise

In each of the questions, 1 to 24, write the correct answer from the given four options.

1. 196 is the square of

(a) 11 (b) 12 (c) 14 (d) 16

2. Which of the following is a square of an even number?

(a) 144 (b) 169 (c) 441 (d) 625

3. A number ending in 9 will have the units place of its square as

(a) 3 (b) 9 (c) 1 (d) 6

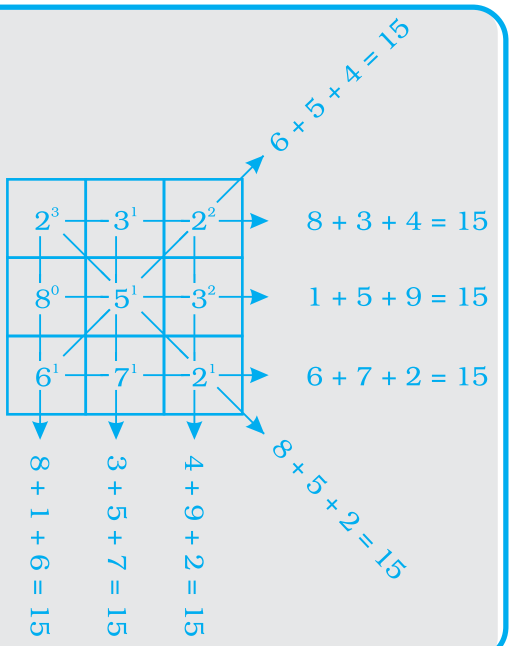
Magic Squares

A magic square is a square with numbers arranged so that the sum of the numbers in each row, column and diagonal is the same.

Complete each magic square below.

$\sqrt{36}$	▢	2^2
8^0	$\sqrt{9}$	▢
▢	$3^2 - 2$	▢

▢	$-(\sqrt{4} + 4)$	$-(9^2)$
$-(\sqrt{16})$	▢	0^3
$-(\sqrt{9})$	$2^0 + 1$	▢



Use the numbers –4, –3, –2, –1, 0, 1, 2, 3 and 4 to make a magic square with row, column and diagonal sums of 0.

4. Which of the following will have 4 at the units place?

(a) 14^2 (b) 62^2 (c) 27^2 (d) 35^2

5. How many natural numbers lie between 5^2 and 6^2?

(a) 9 (b) 10 (c) 11 (d) 12

6. Which of the following cannot be a perfect square?

(a) 841 (b) 529 (c) 198

(d) All of the above

7. The one's digit of the cube of 23 is

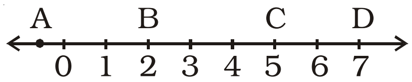
(a) 6 (b) 7 (c) 3 (d) 9

8. A square board has an area of 144 square units. How long is each side of the board?

(a) 11 units (b) 12 units (c) 13 units (d) 14 units

9. Which letter best represents the location of $\sqrt{25}$ on a number line?

(a) A (b) B (c) C (d) D



10. If one member of a pythagorean triplet is 2m, then the other two members are

(a) m, m^2+1

(b) m^2+1, m^2-1

(c) m^2, m^2-1

(d) $m^2, m+1$

11. The sum of successive odd numbers 1, 3, 5, 7, 9, 11, 13 and 15 is

(a) 81 (b) 64 (c) 49 (d) 36

12. The sum of first n odd natural numbers is

(a) $2n+1$ (b) n^2 (c) n^2-1 (d) n^2+1

13. Which of the following numbers is a perfect cube?

(a) 243 (b) 216 (c) 392 (d) 8640

14. The hypotenuse of a right triangle with its legs of lengths $3x \times 4x$ is

(a) $5x$ (b) $7x$ (c) $16x$ (d) $25x$

15. The next two numbers in the number pattern 1, 4, 9, 16, 25 ... are

(a)35, 48 (b) 36, 49 (c) 36, 48 (d) 35,49

16. Which among 43^2, 67^2, 52^2, 59^2 would end with digit 1?

(a) 43^2 (b) 67^2 (c) 52^2 (d) 59^2

17. A perfect square can never have the following digit in its ones place.

(a) 1 (b) 8 (c) 0 (d) 6

18. Which of the following numbers is not a perfect cube?

(a) 216 (b) 567 (c) 125 (d) 343

19. $\sqrt[3]{1000}$ is equal to

(a) 10 (b) 100 (c) 1

(d) None of these

20. If m is the square of a natural number n, then n is

(a) the square of m

(b) greater than m

(c) equal to m

(d) $\sqrt{m}$

21. A perfect square number having n digits where n is even will have square root with

(a) $n+1$ digit (b) $\frac{n}{2}$ digit (c) $\frac{n}{3}$ digit (d) $\frac{n+1}{2}$ digit

22. If m is the cube root of n, then n is

(a) m^3 (b) $\sqrt{m}$ (c) $\frac{m}{3}$ (d) $\sqrt[3]{m}$

23. The value of $\sqrt{248+\sqrt{52+\sqrt{144}}}$ is

(a) 14 (b) 12 (c) 16 (d) 13

24. Given that $\sqrt{4096} = 64$, the value of $\sqrt{4096}+\sqrt{40.96}$ is

(a) 74 (b) 60.4 (c) 64.4 (d) 70.4

In questions 25 to 48, fill in the blanks to make the statements true.

25. There are _________ perfect squares between 1 and 100.

26. There are _________ perfect cubes between 1 and 1000.

27. The units digit in the square of 1294 is _________.

28. The square of 500 will have _________ zeroes.

29. There are _________ natural numbers between n^2 and $(n + 1)^2$

30. The square root of 24025 will have _________ digits.

31. The square of 5.5 is _________.

32. The square root of 5.3×5.3 is _________.

33. The cube of 100 will have _________ zeroes.

34. 1m^2 = _________ cm^2.

35. 1m^3 = _________ cm^3.

36. Ones digit in the cube of 38 is _________.

37. The square of 0.7 is _________.

38. The sum of first six odd natural numbers is _________.

39. The digit at the ones place of 57^2 is _________.

40. The sides of a right triangle whose hypotenuse is 17cm are _________ and _________.

41. $\sqrt{1.96}$ = _________.

42. $(1.2)^3$ = _________.

43. The cube of an odd number is always an _________ number.

44. The cube root of a number x is denoted by _________.

45. The least number by which 125 be multiplied to make it a perfect square is _____________.

46. The least number by which 72 be multiplied to make it a perfect cube is _____________.

47. The least number by which 72 be divided to make it a perfect cube is _____________.

48. Cube of a number ending in 7 will end in the digit _______________.

In questions 49 to 86, state whether the statements are true (T) or false (F).

49. The square of 86 will have 6 at the units place.

50. The sum of two perfect squares is a perfect square.

51. The product of two perfect squares is a perfect square.

52. There is no square number between 50 and 60.

53. The square root of 1521 is 31.

54. Each prime factor appears 3 times in its cube.

55. The square of 2.8 is 78.4.

56. The cube of 0.4 is 0.064.

57. The square root of 0.9 is 0.3.

58. The square of every natural number is always greater than the number itself.

59. The cube root of 8000 is 200.

60. There are five perfect cubes between 1 and 100.

61. There are 200 natural numbers between 100^2 and 101^2.

62. The sum of first n odd natural numbers is n^2.

63. 1000 is a perfect square.

64. A perfect square can have 8 as its units digit.

65. For every natural number m, $(2m–1, 2m^2–2m, 2m^2–2m + 1)$ is a pythagorean triplet.

66. All numbers of a pythagorean triplet are odd.

67. For an integer a, a^3 is always greater than a^2.

68. If x and y are integers such that $x^2 > y^2$, then $x^3 > y^3$.

69. Let x and y be natural numbers. If x divides y, then x^3 divides y^3.

70. If a^2 ends in 5, then a^3 ends in 25.

71. If a^2 ends in 9, then a^3 ends in 7.

72. The square root of a perfect square of n digits will have $\left(\frac{n+1}{2}\right)$ digits, if n is odd.

73. Square root of a number x is denoted by $\sqrt{x}$.

74. A number having 7 at its ones place will have 3 at the units place of its square.

What's the Error? A student said that since the square roots of a certain number are 1.5 and –1.5, the number must be their product, –2.25. What error did the student make?

75. A number having 7 at its ones place will have 3 at the ones place of its cube.

76. The cube of a one digit number cannot be a two digit number.

77. Cube of an even number is odd.

78. Cube of an odd number is even.

79. Cube of an even number is even.

80. Cube of an odd number is odd.

81. 999 is a perfect cube.

82. 363 × 81 is a perfect cube.

83. Cube roots of 8 are +2 and –2.

84. $\sqrt[3]{8+27} = \sqrt[3]{8} + \sqrt[3]{27}$.

85. There is no cube root of a negative integer.

86. Square of a number is positive, so the cube of that number will also be positive.

Solve the following questions.

87. Write the first five square numbers.

88. Write cubes of first three multiples of 3.

89. Show that 500 is not a perfect square.

90. Express 81 as the sum of first nine consecutive odd numbers.

91. Using prime factorisation, find which of the following are perfect squares.

(a)484 (b) 11250 (c) 841 (d) 729

92. Using prime factorisation, find which of the following are perfect cubes.

(a)128 (b) 343 (c) 729 (d) 1331

93. Using distributive law, find the squares of

(a)101 (b) 72

94. Can a right triangle with sides 6cm, 10cm and 8cm be formed? Give reason.

95. Write the Pythagorean triplet whose one of the numbers is 4.

96. Using prime factorisation, find the square roots of

(a)11025 (b) 4761

97. Using prime factorisation, find the cube roots of

(a)512 (b) 2197

98. Is 176 a perfect square? If not, find the smallest number by which it should be multiplied to get a perfect square.

99. Is 9720 a perfect cube? If not, find the smallest number by which it should be divided to get a perfect cube.

100. Write two Pythagorean triplets each having one of the numbers as 5.

101. By what smallest number should 216 be divided so that the quotient is a perfect square. Also find the square root of the quotient.

102. By what smallest number should 3600 be multiplied so that the quotient is a perfect cube. Also find the cube root of the quotient.

103. Find the square root of the following by long division method.

(a)1369 (b) 5625

104. Find the square root of the following by long division method.

(a)27.04 (b) 1.44

105. What is the least number that should be subtracted from 1385 to get a perfect square? Also find the square root of the perfect square.

106. What is the least number that should be added to 6200 to make it a perfect square?

107. Find the least number of four digits that is a perfect square.

108. Find the greatest number of three digits that is a perfect square.

109. Find the least square number which is exactly divisible by 3, 4, 5, 6 and 8.

110. Find the length of the side of a square if the length of its diagonal is 10cm.

111. A decimal number is multiplied by itself. If the product is 51.84, find the number.

112. Find the decimal fraction which when multiplied by itself gives 84.64.

113. A farmer wants to plough his square field of side 150m. How much area will he have to plough?

114. What will be the number of unit squares on each side of a square graph paper if the total number of unit squares is 256?

115. If one side of a cube is 15m in length, find its volume.

116. The dimensions of a rectangular field are 80m and 18m. Find the length of its diagonal.

117. Find the area of a square field if its perimeter is 96m.

118. Find the length of each side of a cube if its volume is 512 cm^3.

119. Three numbers are in the ratio 1:2:3 and the sum of their cubes is 4500. Find the numbers.

120. How many square metres of carpet will be required for a square room of side 6.5m to be carpeted.

121. Find the side of a square whose area is equal to the area of a rectangle with sides 6.4m and 2.5m.

122. Difference of two perfect cubes is 189. If the cube root of the smaller of the two numbers is 3, find the cube root of the larger number.

123. Find the number of plants in each row if 1024 plants are arranged so that number of plants in a row is the same as the number of rows.

124. A hall has a capacity of 2704 seats. If the number of rows is equal to the number of seats in each row, then find the number of seats in each row.

125. A General wishes to draw up his 7500 soldiers in the form of a square. After arranging, he found out that some of them are left out. How many soldiers were left out?

126. 8649 students were sitting in a lecture room in such a manner that there were as many students in the row as there were rows in the lecture room. How many students were there in each row of the lecture room?

127. Rahul walks 12m north from his house and turns west to walk 35m to reach his friend's house. While returning, he walks diagonally from his friend's house to reach back to his house. What distance did he walk while returning?

128. A 5.5m long ladder is leaned against a wall. The ladder reaches the wall to a height of 4.4m. Find the distance between the wall and the foot of the ladder.

129. A king wanted to reward his advisor, a wise man of the kingdom. So he asked the wiseman to name his own reward. The wiseman thanked the king but said that he would ask only for some gold coins each day for a month. The coins were to be counted out in a pattern of one coin for the first day, 3 coins for the second day, 5 coins for the third day and so on for 30 days. Without making calculations, find how many coins will the advisor get in that month?

130. Find three numbers in the ratio 2:3:5, the sum of whose squares is 608.

131. Find the smallest square number divisible by each one of the numbers 8, 9 and 10.

132. The area of a square plot is $101\frac{1}{400}$ m^2. Find the length of one side of the plot.

133. Find the square root of 324 by the method of repeated subtraction.

134. Three numbers are in the ratio 2:3:4. The sum of their cubes is 0.334125. Find the numbers.

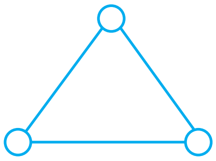
135. Evaluate : $\sqrt[3]{27}+\sqrt[3]{0.008}+\sqrt[3]{0.064}$

136. $\left\{\left(5^2+\left(12^2\right)^{\frac{1}{2}}\right)\right\}^3$

137. $\left\{\left(6^2+\left(8^2\right)^{\frac{1}{2}}\right)\right\}^3$

138. A perfect square number has four digits, none of which is zero. The digits from left to right have values that are: even, even, odd, even. Find the number.

139. Put three different numbers in the circles so that when you add the numbers at the end of each line you always get a perfect square.



140. The perimeters of two squares are 40 and 96 metres respectively. Find the perimeter of another square equal in area to the sum of the first two squares.

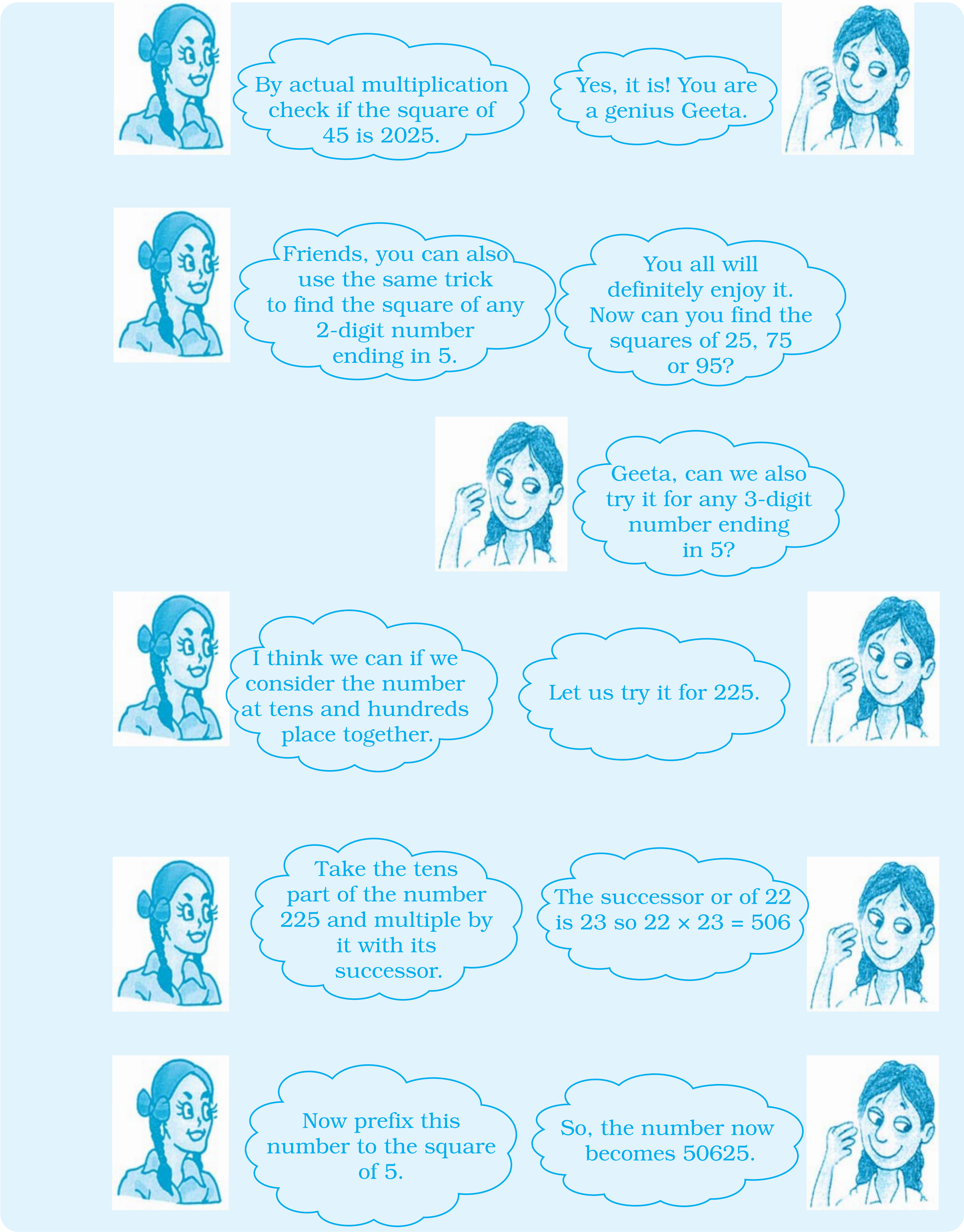
141. A three digit perfect square is such that if it is viewed upside down, the number seen is also a perfect square. What is the number?

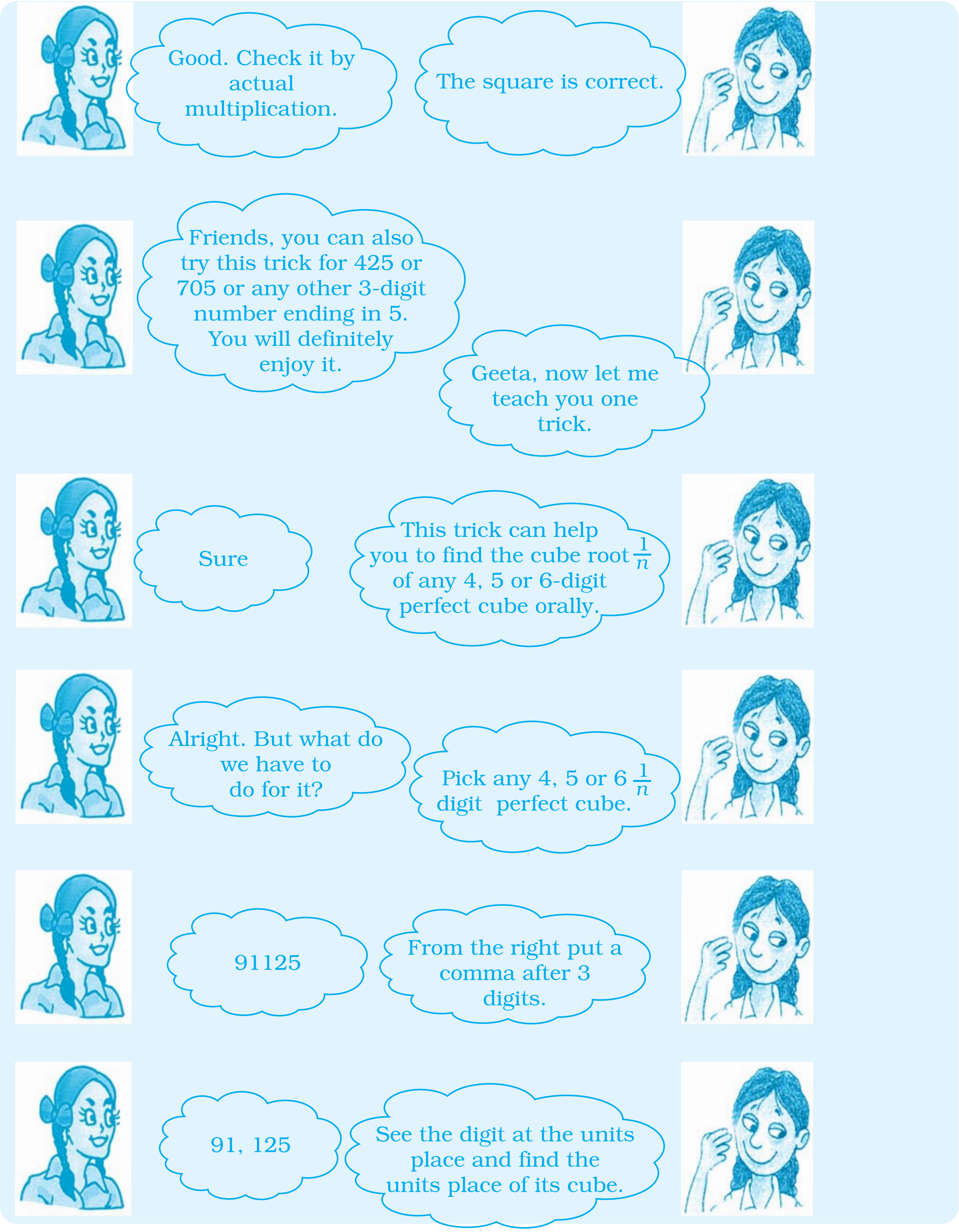
(Hint: The digits 1, 0 and 8 stay the same when viewed upside down, whereas 9 becomes 6 and 6 becomes 9.)

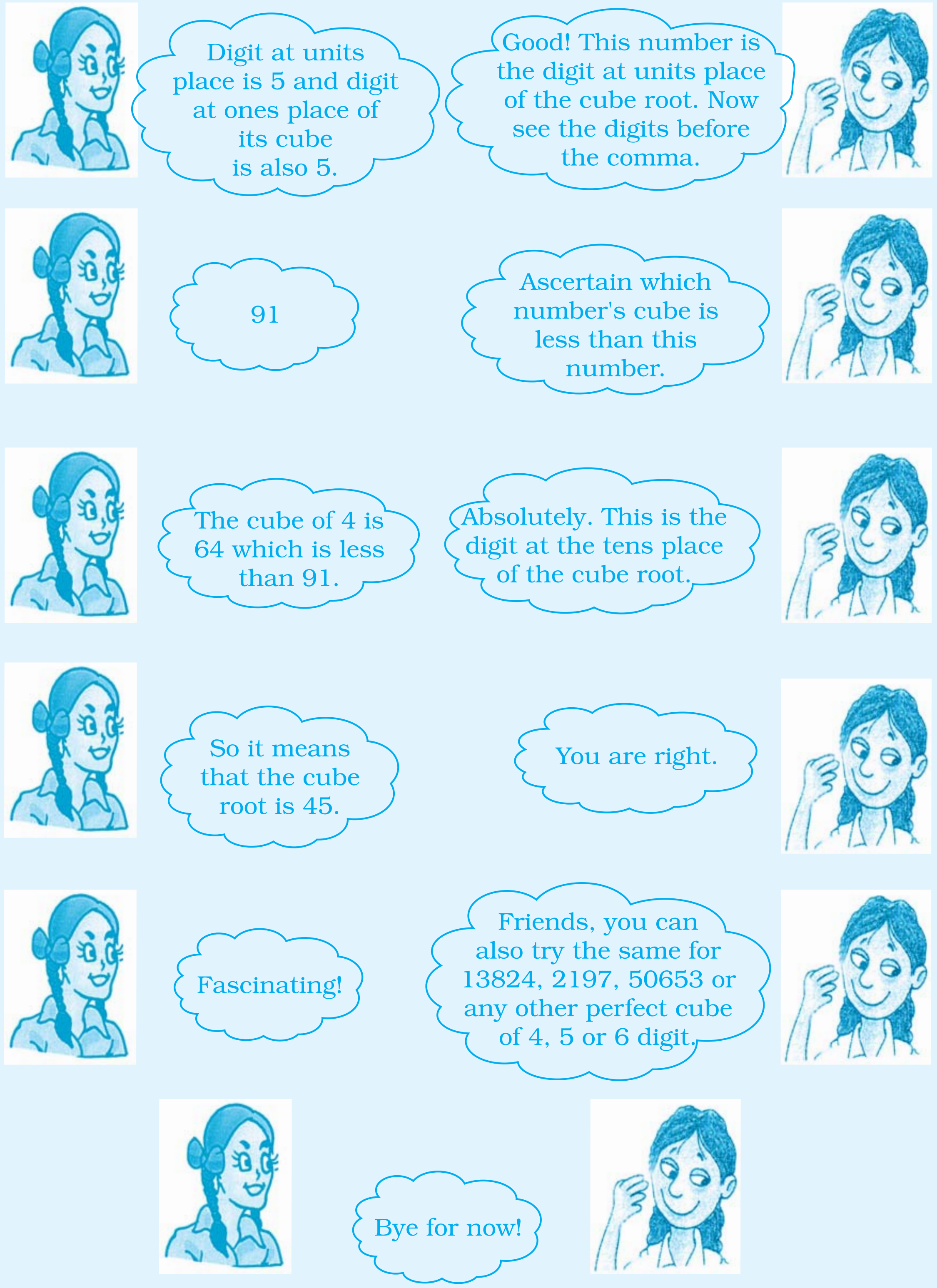
142. 13 and 31 is a strange pair of numbers such that their squares 169 and 961 are also mirror images of each other. Can you find two other such pairs?

(D) Applications, Games and Puzzles

1. Quick Tricks.


By actual multiplication check if the square of 45 is 2025.
Yes, it is! You are a genius Geeta.
Friends, you can also use the same trick to find the square of any 2-digit number ending in 5.
You all will definitely enjoy it. Now can you find the squares of 25, 75 or 95?
Geeta, can we also try it for any 3-digit number ending in 5?
I think we can if we consider the number at tens and hundreds place together.
Let us try it for 225.
Take the tens part of the number 225 and multiple by it with its successor.
The successor or of 22 is 23 so 22 × 23 = 506
Now prefix this number to the square of 5.
So, the number now becomes 50625.


Good. Check it by actual multiplication.
The square is correct.
Friends, you can also try this trick for 425 or 705 or any other 3-digit number ending in 5. You will definitely enjoy it.
Geeta, now let me teach you one trick.
Sure
This trick can help you to find the cube root $\frac{1}{n}$ of any 4, 5 or 6-digit perfect cube orally.
Alright. But what do we have to do for it?
Pick any 4, 5 or 6 $\frac{1}{n}$ digit perfect cube.
91125
From the right put a comma after 3 digits.
91, 125
See the digit at the units place and find the units place of its cube.


Digit at units place is 5 and digit at ones place of its cube is also 5.
Good! This number is the digit at units place of the cube root. Now see the digits before the comma.
91
Ascertain which number's cube is less than this number.
The cube of 4 is 64 which is less than 91.
Absolutely. This is the digit at the tens place of the cube root.
So it means that the cube root is 45.
You are right.
Fascinating!
Friends, you can also try the same for 13824, 2197, 50653 or any other perfect cube of 4, 5 or 6 digit.
Bye for now!

2. Cross Number Puzzle

Down

1. Cube of 9.
2. Missing number to make 12, ____, 37, a pythagorean triplet.
4. Smallest number by which 248 be multiplied to make the resultant a perfect cube number.
5. Square of 75.
6. Smallest square number that is divisible by each of 5 and 11
9. Without adding, find the sum of 1 + 3 + 5 + 7 + 9 + 11.
10. Smallest number which when added to 7669 makes the resultant a perfect square.

Across

2. Square of 19.
3. Look at the numbers given below and find the number which cannot be a perfect square.

   81, 100, 144, 25000
7. Square root of 4489
8. Smallest natural number other than 1 which is a perfect square as well as a perfect cube number.
10. Cube root of 357911.
11. Smallest number which when subtracted from 374695 makes the resultant a perfect square number.

1	2			6
3	5			
4	7	10		
			9	
11			8	

Rough Work

Rough Work

Rough Work