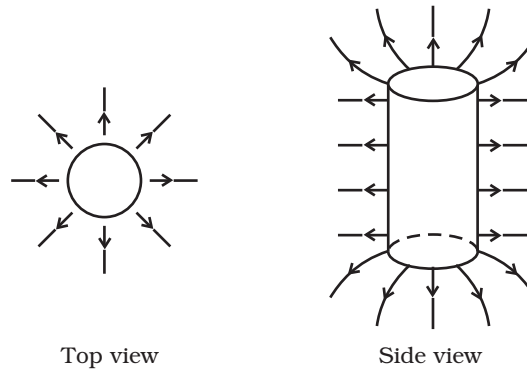


ANSWERS

Chapter 1

- 1.1 (a)
1.2 (a)
1.3 (d)
1.4 (b)
1.5 (c)
1.6 (a)
1.7 (a)
1.8 (c), (d)
1.9 (b), (d)
1.10 (b), (d)
1.11 (c), (d)
1.12 (a), (c).
1.13 (a), (b), (c) and (d).
1.14 Zero.
1.15 (i) $\frac{-Q}{4\pi R_1^2}$ (ii) $\frac{Q}{4\pi R_2^2}$
1.16 The electric fields bind the atoms to neutral entity. Fields are caused by excess charges. There can be no excess charge on the inter surface of an isolated conductor.
1.17 No, the field may be normal. However, the converse is true.

1.18



1.19 (i) $\frac{q}{8\epsilon_0}$ (ii) $\frac{q}{4\epsilon_0}$ (iii) $\frac{q}{2\epsilon_0}$ (iv) $\frac{q}{2\epsilon_0}$

1.20 1 Molar mass M of Al has $N_A = 6.023 \times 10^{23}$ atoms.

$\therefore m =$ mass of Al paisa coin has $N = N_A \frac{m}{M}$ atoms

Now, $Z_{Al} = 13$, $M_{Al} = 26.9815\text{g}$

Hence $N = 6.02 \times 10^{23} \text{ atoms/mol} \times \frac{0.75}{26.9815\text{g/mol}}$
 $= 1.6733 \times 10^{22}$ atoms

$\therefore q =$ +ve charge in paisa $= N Ze$
 $= (1.67 \times 10^{22})(13) (1.60 \times 10^{-19}\text{C})$
 $= 3.48 \times 10^4 \text{ C.}$

$q = 34.8 \text{ kC}$ of \pm ve charge.

This is an enormous amount of charge. Thus we see that ordinary neutral matter contains enormous amount of \pm charges.

1.21 (i) $F_1 = \frac{|q|^2}{4\pi\epsilon_0 r_1^2} = \left(8.99 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}\right) \frac{(3.48 \times 10^4 \text{C})^2}{10^{-4}\text{m}^2} = 1.1 \times 10^{23} \text{N}$

(ii) $\frac{F_2}{F_1} = \frac{r_1^2}{r_2^2} = \frac{(10^{-2}\text{m})^2}{(10^2\text{m})^2} = 10^{-8} \Rightarrow F_2 = F_1 \times 10^{-8} = 1.1 \times 10^{15} \text{N}$

(iii) $\frac{F_3}{F_1} = \frac{r_1^2}{r_3^2} = \frac{(10^{-2}\text{m})^2}{(10^6\text{m})^2} = 10^{-16}$

$F_3 = 10^{-16} F_1 = 1.1 \times 10^7 \text{N.}$

Conclusion: When separated as point charges these charges exert an enormous force. It is not easy to disturb electrical neutrality.

Exemplar Problems–Physics

- 1.22 (i) Zero, from symmetry.
 (ii) Removing a +ve Cs ion is equivalent to adding singly charged -ve Cs ion at that location.
 Net force then is

$$F = \frac{e^2}{4\pi\epsilon_0 r^2}$$

where r = distance between the Cl ion and a Cs ion.

$$\begin{aligned} &= \sqrt{(0.20)^2 + (0.20)^2 + (0.20)^2} \times 10^{-9} = \sqrt{3(0.20)^2} \times 10^{-9} \\ &= 0.346 \times 10^{-9} \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Hence, } F &= \frac{(8.99 \times 10^9)(1.6 \times 10^{-19})^2}{(0.346 \times 10^{-9})^2} = 192 \times 10^{-11} \\ &= 1.92 \times 10^{-9} \text{ N} \end{aligned}$$

Ans 1.92×10^{-9} N, directed from A to Cl^-

- 1.23 At P: on $2q$, Force due to q is to the left and that due to $-3q$ is to the right.

$$\therefore \frac{2q^2}{4\pi\epsilon_0 x^2} = \frac{6q^2}{4\pi\epsilon_0 (d+x)^2}$$

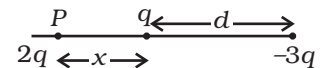
$$\therefore (d+x)^2 = 3x^2$$

$$\therefore 2x^2 - 2dx - d^2 = 0$$

$$x = \frac{d}{2} \pm \frac{\sqrt{3}d}{2}$$

(-ve sign would be between q and $-3q$ and hence is unacceptable.)

$$x = \frac{d}{2} + \frac{\sqrt{3}d}{2} = \frac{d}{2}(1 + \sqrt{3}) \text{ to the left of } q.$$



- 1.24 (a) Charges A and C are positive since lines of force emanate from them.
 (b) Charge C has the largest magnitude since maximum number of field lines are associated with it.
 (c) (i) near A. There is no neutral point between a positive and a negative charge. A neutral point may exist between two like charges. From the figure we see that a neutral point exists between charges A and C. Also between two like charges the neutral point is closer to the charge with smaller magnitude. Thus, electric field is zero near charge A.

- 1.25 (a) (i) zero (ii) $\frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$ along \overline{OA} (iii) $\frac{1}{4\pi\epsilon_0} \frac{2q}{r^2}$ along \overline{OA}

- (b) same as (a).

- 1.26 (a) Let the Universe have a radius R . Assume that the hydrogen atoms are uniformly distributed. The charge on each hydrogen atom is $e_H = -(1 + y)e + e = -ye = |ye|$

The mass of each hydrogen atom is $\sim m_p$ (mass of proton). Expansion starts if the Coulomb repulsion on a hydrogen atom, at R , is larger than the gravitational attraction.

Let the Electric Field at R be \mathbf{E} . Then

$$4\pi R^2 E = \frac{4}{3\epsilon_0} \pi R^3 N |ye| \quad (\text{Gauss's law})$$

$$\mathbf{E}(R) = \frac{1}{3} \frac{N |ye|}{\epsilon_0} R \hat{\mathbf{r}}$$

Let the gravitational field at R be G_R . Then

$$-4\pi R^2 G_R = 4 \pi G m_p \frac{4}{3} \pi R^3 N$$

$$G_R = -\frac{4}{3} \pi G m_p N R$$

$$\mathbf{G}_R(\mathbf{R}) = -\frac{4}{3} \pi G m_p N R \hat{\mathbf{r}}$$

Thus the Coulombic force on a hydrogen atom at R is

$$ye\mathbf{E}(R) = \frac{1}{3} \frac{Ny^2e^2}{\epsilon_0} R \hat{\mathbf{r}}$$

The gravitational force on this atom is

$$m_p \mathbf{G}_R(R) = -\frac{4\pi}{3} GNm_p^2 R \hat{\mathbf{r}}$$

The net force on the atom is

$$\mathbf{F} = \left(\frac{1}{3} \frac{Ny^2e^2}{\epsilon_0} R - \frac{4\pi}{3} GNm_p^2 R \right) \hat{\mathbf{r}}$$

The critical value is when

$$\frac{1}{3} \frac{Ny_c^2e^2}{\epsilon_0} R = \frac{4\pi}{3} GNm_p^2 R$$

$$\Rightarrow y_c^2 = 4\pi\epsilon_0 G \frac{m_p^2}{e^2}$$

$$\square \frac{7 \times 10^{-11} \times 1.8^2 \times 10^6 \times 81 \times 10^{-62}}{9 \times 10^9 \times 1.6^2 \times 10^{-38}}$$

$$\square 63 \times 10^{-38}$$

$$\therefore y_c \square 8 \times 10^{-19} \square 10^{-18}$$

- (b) Because of the net force, the hydrogen atom experiences an acceleration such that

$$m_p \frac{d^2 R}{dt^2} = \left(\frac{1}{3} \frac{Ny^2 e^2}{\epsilon_0} R - \frac{4p}{3} GNm_p^2 R \right)$$

$$\text{Or, } \frac{d^2 R}{dt^2} = \alpha^2 R \text{ where } \alpha^2 = \frac{1}{m_p} \left(\frac{1}{3} \frac{Ny^2 e^2}{\epsilon_0} - \frac{4p}{3} GNm_p^2 \right)$$

This has a solution $R = Ae^{\alpha t} + Be^{-\alpha t}$

As we are seeking an expansion, $B = 0$.

$$\therefore R = Ae^{\alpha t}$$

$$\Rightarrow \dot{R} = \alpha Ae^{\alpha t} = \alpha R$$

Thus, the velocity is proportional to the distance from the centre.

- 1.27 (a) The symmetry of the problem suggests that the electric field is radial. For points $r < R$, consider a spherical Gaussian surfaces. Then on the surface

$$\oiint \mathbf{E}_r \cdot d\mathbf{S} = \frac{1}{\epsilon_0} \int_V \rho dv$$

$$\begin{aligned} 4\pi r^2 E_r &= \frac{1}{\epsilon_0} 4\pi k \int_0^r r'^3 dr' \\ &= \frac{1}{\epsilon_0} \frac{4\pi k}{4} r^4 \end{aligned}$$

$$\therefore E_r = \frac{1}{4\epsilon_0} kr^2$$

$$\mathbf{E}(r) = \frac{1}{4\epsilon_0} kr^2 \hat{\mathbf{r}}$$

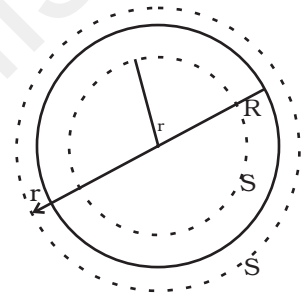
For points $r > R$, consider a spherical Gaussian surfaces' of radius r ,

$$\oiint \mathbf{E}_r \cdot d\mathbf{S} = \frac{1}{\epsilon_0} \int_V \rho dv$$

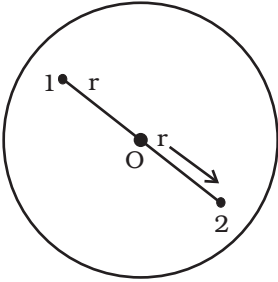
$$\begin{aligned} 4\pi r^2 E_r &= \frac{4\pi k}{\epsilon_0} \int_0^R r^3 dr \\ &= \frac{4\pi k}{\epsilon_0} \frac{R^4}{4} \end{aligned}$$

$$\therefore E_r = \frac{k}{4\epsilon_0} \frac{R^4}{r^2}$$

$$\mathbf{E}(r) = (k/4\epsilon_0) (R^4 / r^2) \hat{\mathbf{r}}$$



- (b) The two protons must be on the opposite sides of the centre along a diameter. Suppose the protons are at a distance r from the centre.



$$\text{Now, } 4\pi \int_0^R kr^3 dr = 2e$$

$$\therefore \frac{4\pi k}{4} R^4 = 2e$$

$$\therefore k = \frac{2e}{\pi R^4}$$

Consider the forces on proton 1. The attractive force due to the charge distribution is

$$-e\mathbf{E}_r = -\frac{e}{4\epsilon_0} k r^2 \hat{\mathbf{r}} = -\frac{2e^2}{4\pi\epsilon_0 R^4} r^2 \hat{\mathbf{r}}$$

$$\text{The repulsive force is } \frac{e^2}{4\pi\epsilon_0 (2r)^2} \hat{\mathbf{r}}$$

$$\text{Net force is } \left(\frac{e^2}{4\pi\epsilon_0 4r^2} - \frac{2e^2}{4\pi\epsilon_0 R^4} r^2 \right) \hat{\mathbf{r}}$$

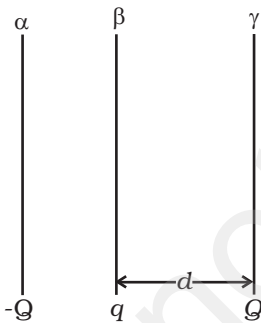
This is zero such that

$$\frac{e^2}{16\pi\epsilon_0 r^2} = \frac{2e^2}{4\pi\epsilon_0 R^4} r^2$$

$$\text{Or, } r^4 = \frac{4R^4}{32} = \frac{R^4}{8}$$

$$\Rightarrow r = \frac{R}{(8)^{1/4}}$$

Thus, the protons must be at a distance $r = \frac{R}{\sqrt[4]{8}}$ from the centre.



1.28 (a) The electric field at γ due to plate α is $-\frac{Q}{S2\epsilon_0} \hat{\mathbf{x}}$

$$\text{The electric field at } \gamma \text{ due to plate } \beta \text{ is } \frac{q}{S2\epsilon_0} \hat{\mathbf{x}}$$

Hence, the net electric field is

$$\mathbf{E}_1 = \frac{(Q - q)}{2\epsilon_0 S} (-\hat{\mathbf{x}})$$

- (b) During the collision plates β & γ are together and hence must be at one potential. Suppose the charge on β is q_1 and on γ is q_2 . Consider a point O. The electric field here must be zero.

$$\text{Electric field at O due to } \alpha = -\frac{Q}{2\epsilon_0 S} \hat{\mathbf{x}}$$

Electric field at O due to $\beta = -\frac{q_1}{2\epsilon_0 S} \hat{x}$

Electric Field at O due to $\gamma = -\frac{q_2}{2\epsilon_0 S} \hat{x}$

$$\therefore \frac{-(Q+q_2)}{2\epsilon_0 S} + \frac{q_1}{2\epsilon_0 S} = 0$$

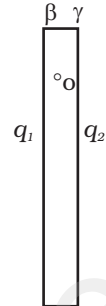
$$\Rightarrow q_1 - q_2 = Q$$

Further, $q_1 + q_2 = Q + q$

$$\Rightarrow q_1 = Q + q/2$$

and $q_2 = q/2$

Thus the charge on β and γ are $Q + q/2$ and $q/2$, respectively.



- (c) Let the velocity be v at the distance d after the collision. If m is the mass of the plate γ , then the gain in K.E. over the round trip must be equal to the work done by the electric field. After the collision, the electric field at γ is

$$\mathbf{E}_2 = -\frac{Q}{2\epsilon_0 S} \hat{x} + \frac{(Q+q/2)}{2\epsilon_0 S} \hat{x} = \frac{q/2}{2\epsilon_0 S} \hat{x}$$

The work done when the plate γ is released till the collision is $F_1 d$ where F_1 is the force on plate γ .

The work done after the collision till it reaches d is $F_2 d$ where F_2 is the force on plate γ .

$$F_1 = E_1 Q = \frac{(Q-q)Q}{2\epsilon_0 S}$$

$$\text{and } F_2 = E_2 q/2 = \frac{(q/2)^2}{2\epsilon_0 S}$$

\therefore Total work done is

$$\frac{1}{2\epsilon_0 S} [(Q-q)Q + (q/2)^2] d = \frac{1}{2\epsilon_0 S} (Q - q/2)^2 d$$

$$\Rightarrow (1/2)mv^2 = \frac{d}{2\epsilon_0 S} (Q - q/2)^2$$

$$\therefore v = (Q - q/2) \left(\frac{d}{m\epsilon_0 S} \right)^{1/2}$$

1.29 (i) $F = \frac{Qq}{r^2} = 1 \text{ dyne} = \frac{[1 \text{ esu of charge}]^2}{[1 \text{ cm}]^2}$

Or,

1 esu of charge = $1 \text{ (dyne)}^{1/2} \text{ (cm)}$

Hence, $[1 \text{ esu of charge}] = [F]^{1/2} L = [MLT^{-2}]^{1/2} L = M^{1/2} L^{3/2} T^{-1}$

$[1 \text{ esu of charge}] = M^{1/2} L^{3/2} T^{-1}$

Thus charge in cgs unit is expressed as fractional powers (1/2) of M and (3/2) of L.

- (ii) Consider the coulomb force on two charges, each of magnitude 1 esu of charge separated by a distance of 1 cm:
 The force is then 1 dyne = 10^{-5} N.
 This situation is equivalent to two charges of magnitude x C separated by 10^{-2} m.
 This gives:

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{x^2}{10^{-4}}$$

which should be 1 dyne = 10^{-5} N. Thus

$$\frac{1}{4\pi\epsilon_0} \cdot \frac{x^2}{10^{-4}} = 10^{-5} \Rightarrow \frac{1}{4\pi\epsilon_0} = \frac{10^{-9} \text{ Nm}^2}{x^2 \text{ C}^2}$$

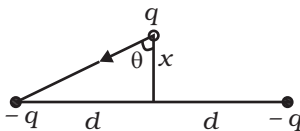
With $x = \frac{1}{[3] \times 10^9}$, this yields

$$\frac{1}{4\pi\epsilon_0} = 10^{-9} \times [3]^2 \times 10^{18} = [3]^2 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}$$

With $[3] \rightarrow 2.99792458$, we get

$$\frac{1}{4\pi\epsilon_0} = 8.98755 \dots \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \text{ exactly}$$

1.30 Net force F on q towards the centre O



$$F = 2 \frac{q^2}{4\pi\epsilon_0 r^2} \cos \theta = - \frac{2q^2}{4\pi\epsilon_0 r^2} \cdot \frac{x}{r}$$

$$F = \frac{-2q^2}{4\pi\epsilon_0} \frac{x}{(d^2 + x^2)^{3/2}}$$

$$\approx \frac{-2q^2}{4\pi\epsilon_0 d^3} x = -k \text{ for } x \ll d.$$

Thus, the force on the third charge q is proportional to the displacement and is towards the centre of the two other charges. Therefore, the motion of the third charge is harmonic with frequency

$$\omega = \sqrt{\frac{2q^2}{4\pi\epsilon_0 d^3 m}} = \sqrt{\frac{k}{m}}$$

$$\text{and hence } T = \frac{2\pi}{\omega} \left[\frac{8\pi^3 \epsilon_0 m d^3}{q^2} \right]^{1/2}.$$

- 1.31 (a) Slight push on q along the axis of the ring gives rise to the situation shown in Fig (b). A and B are two points on the ring at the end of a diameter.

Force on q due to line elements $\frac{-Q}{2\pi R}$ at A and B is

$$F_{A+B} = 2 \cdot \frac{-Q}{2\pi R} \cdot q \cdot \frac{1}{4\pi\epsilon_0} \cdot \frac{1}{r^2} \cdot \cos\theta$$

$$= \frac{-Qq}{\pi R \cdot 4\pi\epsilon_0} \cdot \frac{1}{(z^2 + R^2)} \cdot \frac{z}{(z^2 + R^2)^{1/2}}$$

Total force due to ring on $q = (F_{A+B})(\pi R)$

$$= \frac{-Qq}{4\pi\epsilon_0} \frac{z}{(z^2 + R^2)^{3/2}}$$

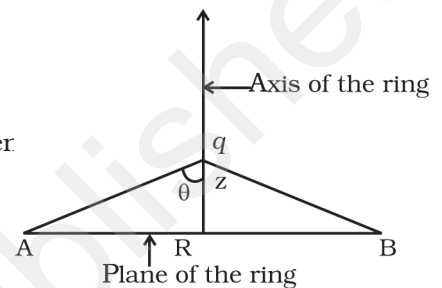
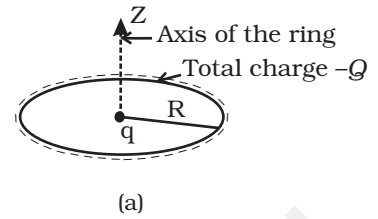
□ $\frac{-Qq}{4\pi\epsilon_0}$ for $z \ll R$

Thus, the force is proportional to negative of displacement under such forces is harmonic.

(b) From (a)

$$m \frac{d^2 z}{dt^2} = -\frac{Qqz}{4\pi\epsilon_0 R^3} \text{ or } \frac{d^2 z}{dt^2} = -\frac{Qq}{4\pi\epsilon_0 m R^3} z$$

That is, $\omega^2 = \frac{Qq}{4\pi\epsilon_0 m R^3}$. Hence $T = 2\pi \sqrt{\frac{4\pi\epsilon_0 m R^3}{Qq}}$



Chapter 2

- 2.1 (d)
- 2.2 (c)
- 2.3 (c)
- 2.4 (c)
- 2.5 (a)
- 2.6 (c)
- 2.7 (b), (c), (d)
- 2.8 (a), (b), (c)
- 2.9 (b), (c)
- 2.10 (b), (c)
- 2.11 (a), (d)
- 2.12 (a), (b)
- 2.13 (c) and (d)

- 2.14 More.
- 2.15 Higher potential.
- 2.16 Yes, if the sizes are different.
- 2.17 No.
- 2.18 As electric field is conservative, work done will be zero in both the cases.
- 2.19 Suppose this were not true. The potential just inside the surface would be different from that at the surface resulting in a potential gradient. This would mean that there are field lines pointing inwards or outwards from the surface. These lines cannot at the other end be again on the surface, since the surface is equipotential. Thus, this is possible only if the other end of the lines are at charges inside, contradicting the premise. Hence, the entire volume inside must be at the same potential.
- 2.20 C will decrease

Energy stored = $\frac{1}{2}CV^2$ and hence will increase.

Electric field will increase.

Charge stored will remain the same.

V will increase.

- 2.21 Consider any path from the charged conductor to the uncharged conductor along the electric field. The potential will continually decrease along this path. A second path from the uncharged conductor to infinity will again continually lower the potential further. Hence this result.

2.22
$$U = \frac{-qQ}{4\pi\epsilon_0 R \sqrt{1+z^2/R^2}}$$

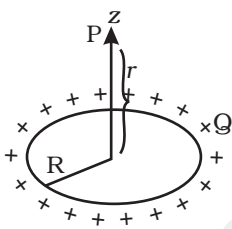
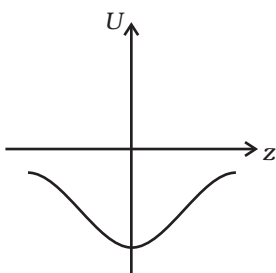
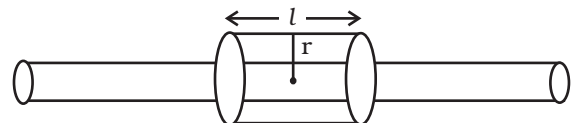
The variation of potential energy with z is shown in the figure.

The charge $-q$ displaced would perform oscillations. We cannot conclude anything just by looking at the graph.

2.23
$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{R^2+z^2}}$$

- 2.24 To find the potential at distance r from the line consider the electric field. We note that from symmetry the field lines must be radially outward. Draw a cylindrical Gaussian surface of radius r and length l . Then

$$\oint \mathbf{E} \cdot d\mathbf{S} = \frac{1}{\epsilon_0} \lambda l$$



Exemplar Problems–Physics

$$\text{Or } E_r 2\pi r l = \frac{1}{\epsilon_0} \lambda l$$

$$\Rightarrow E_r = \frac{\lambda}{2\pi\epsilon_0 r}$$

Hence, if r_0 is the radius,

$$V(r) - V(r_0) = - \int_{r_0}^r \mathbf{E} \cdot d\mathbf{l} = \frac{\lambda}{2\pi\epsilon_0} \ln \frac{r_0}{r}$$

For a given V ,

$$\ln \frac{r}{r_0} = - \frac{2\pi\epsilon_0}{\lambda} [V(r) - V(r_0)]$$

$$\Rightarrow r = r_0 e^{-2\pi\epsilon_0 V(r) / \lambda} e^{+2\pi\epsilon_0 V(r) / \lambda}$$

The equipotential surfaces are cylinders of radius

$$r = r_0 e^{-2\pi\epsilon_0 [V(r) - V(r_0)] / \lambda}$$

- 2.25 Let the plane be at a distance x from the origin. The potential at the point P is

$$\frac{1}{4\pi\epsilon_0} \frac{q}{[(x+d/2)^2 + h^2]^{1/2}} - \frac{1}{4\pi\epsilon_0} \frac{q}{[(x-d/2)^2 + h^2]^{1/2}}$$

If this is to be zero,

$$\frac{1}{[(x+d/2)^2 + h^2]^{1/2}} = \frac{1}{[(x-d/2)^2 + h^2]^{1/2}}$$

$$\text{Or, } (x-d/2)^2 + h^2 = (x+d/2)^2 + h^2$$

$$\Rightarrow x^2 - dx + d^2/4 = x^2 + dx + d^2/4$$

$$\text{Or, } 2dx = 0$$

$$\Rightarrow x = 0$$

The equation is that of a plane $x = 0$.

- 2.26 Let the final voltage be U : If C is the capacitance of the capacitor without the dielectric, then the charge on the capacitor is

$$Q_1 = CU$$

The capacitor with the dielectric has a capacitance ϵC . Hence the charge on the capacitor is

$$Q_2 = \epsilon U = \alpha CU^2$$

The initial charge on the capacitor that was charged is

$$Q_0 = CU_0$$

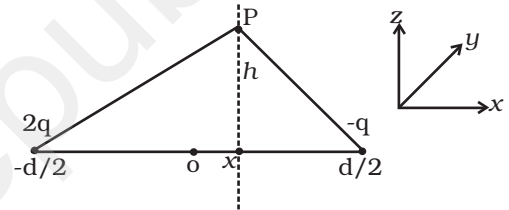
From the conservation of charges,

$$Q_0 = Q_1 + Q_2$$

$$\text{Or, } CU_0 = CU + \alpha CU^2$$

$$\Rightarrow \alpha U^2 + U - U_0 = 0$$

$$\therefore U = \frac{-1 \pm \sqrt{1 + 4\alpha U_0}}{2\alpha}$$



$$= \frac{-1 \pm \sqrt{1+624}}{4}$$

$$= \frac{-1 \pm \sqrt{625}}{4} \text{ volts}$$

As U is positive

$$U = \frac{\sqrt{625} - 1}{4} = \frac{24}{4} = 6V$$

2.27 When the disc is in touch with the bottom plate, the entire plate is a equipotential. A charge q' is transferred to the disc.

The electric field on the disc is

$$= \frac{V}{d}$$

$$\therefore q' = -\epsilon_0 \frac{V}{d} \pi r^2$$

The force acting on the disc is

$$-\frac{V}{d} \times q' = \epsilon_0 \frac{V^2}{d^2} \pi r^2$$

If the disc is to be lifted, then

$$\epsilon_0 \frac{V^2}{d^2} \pi r^2 = mg$$

$$\Rightarrow V = \sqrt{\frac{mgd^2}{\pi\epsilon_0 r^2}}$$

2.28
$$U = \frac{1}{4\pi\epsilon_0} \left\{ \frac{q_d q_d}{r} - \frac{q_u q_d}{r} - \frac{q_u q_d}{r} \right\}$$

$$= \frac{9 \times 10^9}{10^{-15}} (1.6 \times 10^{-19})^2 \left\{ (1/3)^2 - (2/3)(1/3) - (2/3)(1/3) \right\}$$

$$= 2.304 \times 10^{-13} \left\{ \frac{1}{9} - \frac{4}{9} \right\} = -7.68 \times 10^{-14} \text{ J}$$

$$= 4.8 \times 10^5 \text{ eV} = 0.48 \text{ MeV} = 5.11 \times 10^{-4} (m_n c^2)$$

2.29 Before contact

$$Q_1 = \sigma \cdot 4\pi R^2$$

$$Q_2 = \sigma \cdot 4\pi(2R^2) = 4(\sigma \cdot 4\pi R^2) = 4Q_1$$

After contact :

$$Q_1' + Q_2' = Q_1 + Q_2 = 5Q_1,$$

$$= 5(\sigma \cdot 4\pi R^2)$$

Exemplar Problems–Physics

They will be at equal potentials:

$$\frac{Q_1'}{R} = \frac{Q_2'}{2R}$$

$$\therefore Q_2' = 2Q_1'$$

$$\therefore 3Q_1' = 5(\sigma \cdot 4\pi R^2)$$

$$\therefore Q_1' = \frac{5}{3}(\sigma \cdot 4\pi R^2) \text{ and } Q_2' = \frac{10}{3}(\sigma \cdot 4\pi R^2)$$

$$\therefore \sigma_1 = 5/3 \sigma \text{ and } \therefore \sigma_2 = \frac{5}{6} \sigma$$

2.30 Initially : $V \propto \frac{1}{C}$ and $V_1 + V_2 = E$

$$\Rightarrow V_1 = 3V \text{ and } V_2 = 6V$$

$$\therefore Q_1 = C_1 V_1 = 6C \times 3 = 18 \mu C$$

$$Q_2 = 9 \mu C \text{ and } Q_3 = 0$$

$$\text{Later : } Q_2 = Q_2' + Q_3$$

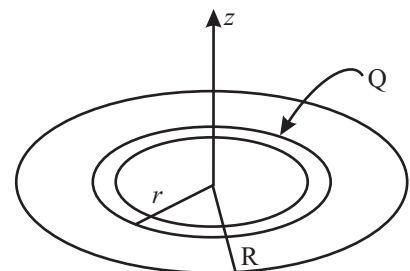
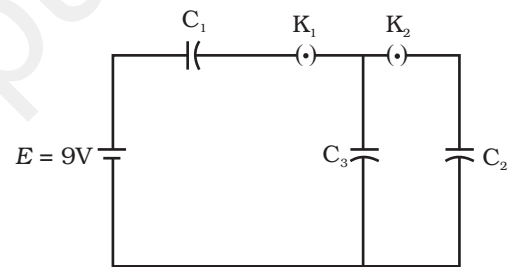
$$\text{with } C_2 V + C_3 V = Q_2 \Rightarrow V = \frac{Q_2}{C_2 + C_3} = (3/2)V$$

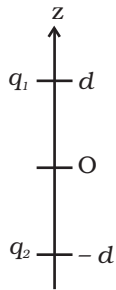
$$Q_2' = (9/2) \mu C \text{ and } Q_3' = (9/2) \mu C$$

2.31 $\sigma = \frac{Q}{\pi R^2}$

$$dU = \frac{1}{4\pi\epsilon_0} \frac{\sigma \cdot 2\pi r dr}{\sqrt{r^2 + z^2}}$$

$$\therefore U = \frac{\pi\sigma}{4\pi\epsilon_0} \int_0^R \frac{2rdr}{\sqrt{r^2 + z^2}}$$





2.32

$$= \frac{2\pi\sigma}{4\pi\epsilon_0} \left[\sqrt{r^2+z^2} \right]_0^R = \frac{2\pi\sigma}{4\pi\epsilon_0} \left[\sqrt{R^2+z^2} - z \right]$$

$$= \frac{2Q}{4\pi\epsilon_0 R^2} \left[\sqrt{R^2+z^2} - z \right]$$

$$\frac{q_1}{\sqrt{x^2+y^2+(z-d)^2}} + \frac{q_2}{\sqrt{x^2+y^2+(z+d)^2}} = 0$$

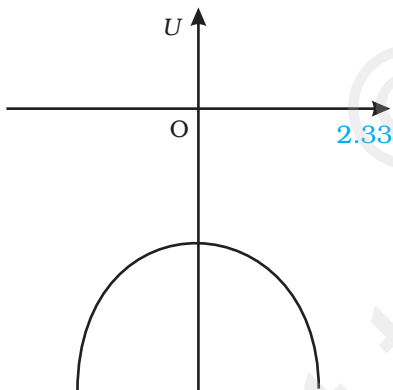
$$\therefore \frac{q_1}{\sqrt{x^2+y^2+(z-d)^2}} = \frac{-q_2}{\sqrt{x^2+y^2+(z+d)^2}}$$

Thus, to have total potential zero, q_1 and q_2 must have opposite signs. Squaring and simplifying, we get.

$$x^2 + y^2 + z^2 + \left[\frac{(q_1/q_2)^2 + 1}{(q_1/q_2)^2 - 1} \right] (2zd) + d^2 = 0$$

This is the equation of a sphere with centre at $\left(0, 0, -2d \left[\frac{q_1^2 + q_2^2}{q_1^2 - q_2^2} \right] \right)$.

Note : if $q_1 = -q_2 \Rightarrow$ Then $z = 0$, which is a plane through mid-point.



2.33

$$U = \frac{1}{4\pi\epsilon_0} \left\{ \frac{-q^2}{(d-x)} + \frac{-q^2}{(d-x)} \right\}$$

$$U = \frac{-q^2}{4\pi\epsilon_0} \frac{2d}{(d^2 - x^2)}$$

$$\frac{dU}{dx} = \frac{-q^2 \cdot 2d}{4\pi\epsilon_0} \cdot \frac{2x}{(d^2 - x^2)^2}$$

$$U_0 = \frac{2q^2}{4\pi\epsilon_0 d} \quad \frac{dU}{dx} = 0 \text{ at } x = 0$$

$x = 0$ is an equilibrium point.

$$\frac{d^2U}{dx^2} = \left(\frac{-2dq^2}{4\pi\epsilon_0} \right) \left[\frac{2}{(d^2 - x^2)^2} - \frac{8x^2}{(d^2 - x^2)^3} \right]$$

$$= \left(\frac{-2dq^2}{4\pi\epsilon_0} \right) \frac{1}{(d^2 - x^2)^3} [2(d^2 - x^2)^2 - 8x^2]$$

At $x = 0$

$$\frac{d^2U}{dx^2} = \left(\frac{-2dq^2}{4\pi\epsilon_0} \right) \left(\frac{1}{d^6} \right) (2d^2), \text{ which is } < 0.$$

Hence, unstable equilibrium.

Chapter 3

3.1 (b)

3.2 (a)

3.3 (c)

3.4 (b)

3.5 (a)

3.6 (a)

3.7 (b), (d)

3.8 (a), (d)

3.9 (a), (b)

3.10 (b), (c)

3.11 (a), (c)

3.12 When an electron approaches a junction, in addition to the uniform \mathbf{E} that it normally faces (which keep the drift velocity \mathbf{v}_d fixed), there are accumulation of charges on the surface of wires at the junction. These produce electric field. These fields alter direction of momentum.

3.13 Relaxation time is bound to depend on velocities of electrons and ions. Applied electric field affects the velocities of electrons by speeds at the order of 1mm/s, an insignificant effect. Change in T , on the other hand, affects velocities at the order of 10^2 m/s. This can affect τ significantly.

[$\rho = \rho(E, T)$ in which E dependence is ignorable for ordinary applied voltages.]

3.14 The advantage of null point method in a Wheatstone bridge is that the resistance of galvanometer does not affect the balance point and there is no need to determine current in resistances and galvanometer and the internal resistance of a galvanometer. R_{unknown} can be calculated

applying Kirchhoff's rules to the circuit. We would need additional accurate measurement of all the currents in resistances and galvanometer and internal resistance of the galvanometer.

3.15 The metal strips have low resistance and need not be counted in the potentiometer length l_1 of the null point. One measures only their lengths along the straight segments (of lengths 1 meter each). This is easily done with the help of centimeter rulings or meter ruler and leads to accurate measurements.

3.16 Two considerations are required: (i) cost of metal, and (ii) good conductivity of metal. Cost factor inhibits silver. Cu and Al are the next best conductors.

3.17 Alloys have low value of temperature co-efficient (less temperature sensitivity) of resistance and high resistivity.

3.18 Power wasted $P_c = I^2 R_c$
where R_c is the resistance of the connecting wires.

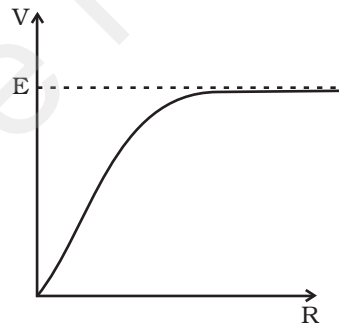
$$P_c = \frac{P^2}{V^2} R_c$$

In order to reduce P_c , power should be transmitted at high voltage.

3.19 If R is increased, the current through the wire will decrease and hence the potential gradient will also decrease, which will result in increase in balance length. So J will shift towards B.

3.20 (i) Positive terminal of E_1 is connected at X and $E_1 > E$.
(ii) Negative terminal of E_1 is connected at X.

3.21



3.22
$$I = \frac{E}{R+nR}; \frac{E}{R+\frac{R}{n}} = 10I$$

$$\frac{1+n}{1+\frac{1}{n}} = 10 = \frac{1+n}{n+1} n = n$$

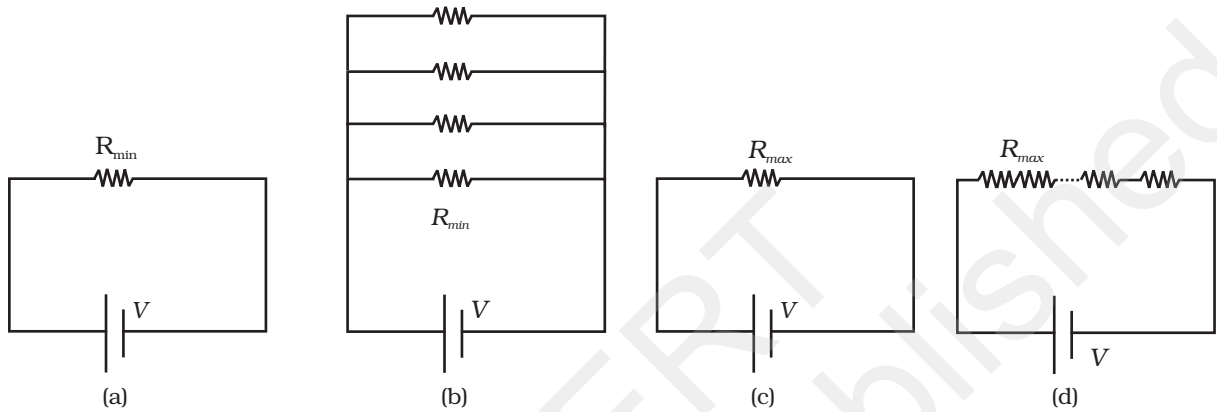
$$\therefore n = 10.$$

3.23
$$\frac{1}{R_p} = \frac{1}{R_1} + \dots + \frac{1}{R_n}, \quad \frac{R_{\min}}{R_p} = \frac{R_{\min}}{R_1} + \frac{R_{\min}}{R_2} + \dots + \frac{R_{\min}}{R_n} > 1$$

Exemplar Problems–Physics

and $R_s = R_1 + \dots + R_n \geq R_{\max}$.

In Fig. (b), R_{\min} provides an equivalent route as in Fig. (a) for current. But in addition there are $(n-1)$ routes by the remaining $(n-1)$ resistors. Current in Fig.(b) > current in Fig. (a). Effective Resistance in Fig. (b) < R_{\min} . Second circuit evidently affords a greater resistance. You can use Fig. (c) and (d) and prove $R_s > R_{\max}$.

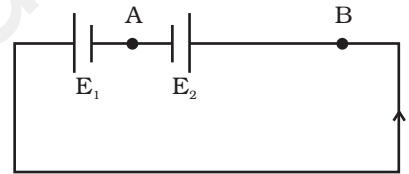


3.24 $I = \frac{6 - 4}{2 + 8} = 0.2A$

P.D. across $E_1 = 6 - 0.2 \times 2 = 5.6 V$

P.D. across $E_2 = V_{AB} = 4 + 0.2 \times 8 = 5.6 V$

Point B is at a higher potential than A



3.25 $I = \frac{E + E}{R + r_1 + r_2}$

$$V_1 = E - Ir_1 = E - \frac{2E}{r_1 + r_2 + R} r_1 = 0$$

$$\text{or } E = \frac{2Er_1}{r_1 + r_2 + R}$$

$$1 = \frac{2r_1}{r_1 + r_2 + R}$$

$$r_1 + r_2 + R = 2r_1$$

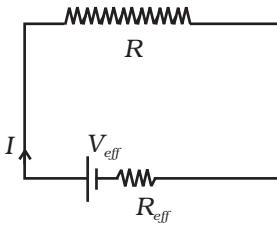
$$R = r_1 - r_2$$

3.26 $R_A = \frac{\rho l}{\pi(10^{-3} \times 0.5)^2}$

$$R_B = \frac{\rho l}{\pi[(10^{-3})^2 - (0.5 \times 10^{-3})^2]}$$

$$\frac{R_A}{R_B} = \frac{(10^{-3})^2 - (0.5 \times 10^{-3})^2}{(0.5 \times 10^{-3})^2} = 3 : 1$$

3.27 We can think of reducing entire network to a simple one for any branch R as shown in Fig.



Then current through R is $I = \frac{V_{eff}}{R_{eff} + R}$

Dimensionally $V_{eff} = V_{eff}(V_1, V_2, \dots, V_n)$ has a dimension of voltage and $R_{eff} = R_{eff}(R_1, R_2, \dots, R_m)$ has a dimension of resistance. Therefore if all are increased n -fold

$$V_{eff}^{new} = nV_{eff}, R_{eff}^{new} = nR_{eff}$$

and $R^{new} = nR$.

Current thus remains the same.

3.28 Applying Kirchhoff's junction rule:

$$I_1 = I + I_2$$

Kirchhoff's loop rule gives:

$$10 = IR + 10I_1 \dots (i)$$

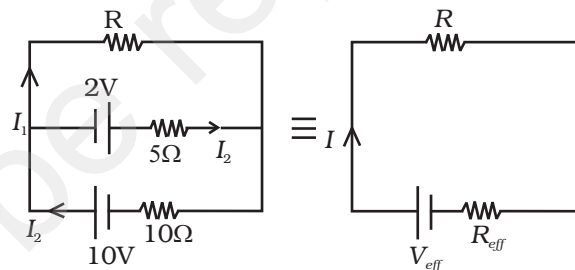
$$2 = 5I_2 - RI = 5(I_1 - I) - RI$$

$$4 = 10I_1 - 10I - 2RI \dots (ii)$$

$$(i) - (ii) \Rightarrow 6 = 3RI + 10I \text{ or, } 2 = I \left(R + \frac{10}{3} \right)$$

$2 = (R + R_{eff})I$ Comparing with $V_{eff} = (R + R_{eff})I$ and $V_{eff} = 2V$

$$R_{eff} = \frac{10}{3} \Omega.$$



3.29 Power consumption = 2units/hour = 2KW = 2000J/s

$$I = \frac{P}{V} = \frac{2000}{220}; 9 \text{ A}$$

Power loss in wire = RI^2 J/s

$$= \rho \frac{l}{A} I^2 = 1.7 \times 10^{-8} \times \frac{10}{\pi \times 10^{-6}} \times 81 \text{ J/s}$$

$$\square 4 \text{ J/s}$$

$$= 0.2\%$$

$$\text{Power loss in Al wire} = 4 \frac{\rho_{Al}}{\rho_{Cu}} = 1.6 \times 4 = 6.4 \text{ J/s} = 0.32\%$$

3.30 Let R' be the resistance of the potentiometer wire.

$$\frac{10 \times R'}{50 + R'} < 8 \Rightarrow 10R' < 400 + 8R'$$

$$2R' < 400 \text{ or } R' < 200\Omega.$$

$$\frac{10 \times R'}{10 + R'} > 8 \Rightarrow 2R' > 80 \Rightarrow R' > 40$$

$$\frac{10 \times \frac{3}{4}R'}{10 + R'} < 8 \Rightarrow 7.5R' < 80 + 8R'$$

$$R' > 160 \Rightarrow 160 < R' < 200.$$

Any R' between 160Ω and 200Ω will achieve.

Potential drop across 400 cm of wire $> 8V$.

Potential drop across 300 cm of wire $< 8V$.

$$\phi \times 400 > 8V \text{ (}\phi \rightarrow \text{potential gradient)}$$

$$\phi \times 300 < 8V$$

$$\phi > 2V/m$$

$$< 2\frac{2}{3} V/m.$$

3.31 (a) $I = \frac{6}{6} = 1 \text{ A} = nev_d A$

$$v_d = \frac{1}{10^{29} \times 1.6 \times 10^{-19} \times 10^{-6}} = \frac{1}{1.6} \times 10^{-4} \text{ m/s}$$

$$\begin{aligned} K.E &= \frac{1}{2} m_e v_d^2 \times nAl \\ &= \frac{1}{2} \times 9.1 \times 10^{-31} \times \frac{1}{2.56} \times 10^{-8} \times 10^{29} \times 10^{-6} \times 10^{-1}; 2 \times 10^{-17} \text{ J} \end{aligned}$$

(b) Ohmic loss = $RI^2 = 6 \times 1^2 = 6 \text{ J/s}$

All of KE of electrons would be lost in $\frac{2 \times 10^{-17}}{6} \text{ s}; 10^{-17} \text{ s}$

Chapter 4

4.1 (d)

4.2 (a)

4.3 (a)

4.4 (d)

4.5 (a)

4.6 (d)

4.7 (a), (b)

4.8 (b), (d)

4.9 (b), (c)

4.10 (b), (c), (d)

4.11 (a), (b), (d)

4.12 For a charge particle moving perpendicular to the magnetic field:

$$\frac{mv^2}{R} = qvB$$

$$\therefore \frac{qB}{m} = \frac{v}{R} = \omega$$

$$\therefore [\omega] = \left[\frac{qB}{m} \right] = \left[\frac{v}{R} \right] = [T]^{-1}.$$

4.13 $dW = \mathbf{F} \cdot d\mathbf{l} = 0$

$$\Rightarrow \mathbf{F} \cdot \mathbf{v} dt = 0$$

$$\Rightarrow \mathbf{F} \cdot \mathbf{v} = 0$$

\mathbf{F} must be velocity dependent which implies that angle between \mathbf{F} and \mathbf{v} is 90° . If \mathbf{v} changes (direction) then (directions) \mathbf{F} should also change so that above condition is satisfied.

4.14 Magnetic force is frame dependent. Net acceleration arising from this is however frame independent (non-relativistic physics) for inertial frames.

4.15 Particle will accelerate and decelerate alternatively. So the radius of path in the Dees will remain unchanged.

4.16 At O_2 , the magnetic field due to I_1 is along the y -axis. The second wire is along the y -axis and hence the force is zero.

4.17 $\mathbf{B} = \frac{1}{4}(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) \frac{\mu_0 I}{2R}$

4.18 No dimensionless quantity $[T]^{-1} = [\omega] = \left[\frac{eB}{m} \right]$

4.19 $\mathbf{E} = E_0 \hat{\mathbf{i}}, E_0 > 0, \mathbf{B} = B_0 \hat{\mathbf{k}}$

4.20 Force due to $d\mathbf{l}_2$ on $d\mathbf{l}_1$ is zero.

Force due to $d\mathbf{l}_1$ on $d\mathbf{l}_2$ is non-zero.

