vè;k; **11**

f=k&foeh; T;kfefr
(Three Dimensional Geometry)

❖ *The moving power of mathematical invention is not reasoning but imagination. – A.DEMORGAN* ❖

11.1 Hkfedk (Introduction)

Leonhard Euler
(1707-1783)

d{kk XI e] o'yf"kd T;kfefr dk vè;;u djr le; f}&foeh; vkj f=k&foeh; fo"k;k oQ ifjp; e geu Lo; dk oQoy dkrh; fof/ rd lhfer j[kk gA bl iLrd oQ fiNy vè;k; e geu lfn'kk dh ey ldYiukvk dk vè;;u fd;k gA vc ge lfn'kk oQ chtxf.kr dk f=k&foeh; T;kfefr e mi;kx djxA f=k&foeh; T;kfefr e bl mikxe dk mnn'; g fd ;g blo vè;;u dk vR;r ljy ,o l#fpi.k (lxkg;) cuk nrk gA*

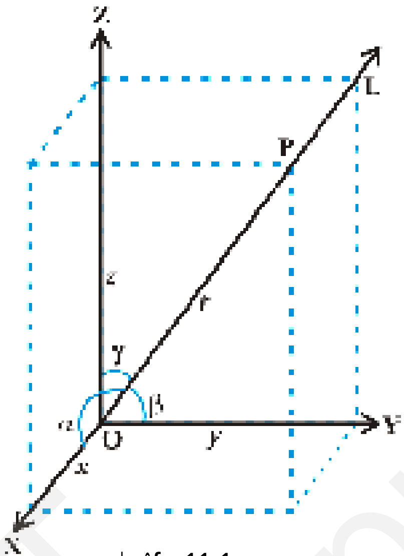
bl vè;k; e ge nk ¯cnvk dk feykus okyh j[kk oQ fnd&dkT;k o fnoQ&vuikr dk vè;;u djx vkj fofHkUu fLFkfr;k e vrfj{k e j[kvk vkj ryk oQ lehdj.kk] nk j[kvk] nk ryk o ,d j[kk vkj ,d ry oQ chp dk dk.k] nk fo"keryh; j[kvk oQ chp U;ure njh o ,d ry dh ,d ¯cn l njh oQ fo"k; e Hkh fopkj foe'k djxA mijkDr ifj.kkek e l vf/dk'k ifj.kkek dk lfn'kk oQ :i e ikIr djr gA rFkfi ge budk dkrh; :i e Hkh vuokn djx tk dkykrj e fLFkfr dk Li"V T;kferh; vkj fo'y"k.kkRed fp=k.k iLrr dj lo\xA

11.2 j[kk oQ fnoQ&dkIkbu vkj fnoQ&vuikr (Direction Cosines and Direction Ratios of a Line)

vè;k; 10 e] Lej.k dhft,] fd ey ¯cn l xtju okyh lfn'k j[kk L }kjk x, y vkj z-v{kk oQ lkFk Øe'k α] β vkj γ cuk, x, dk.k fnoQ&dk.k dgykrg rc bu dk.kk dh dkIkbu uker% $\cos\alpha$, $\cos\beta$ vkj $\cos\gamma$ j[kk L oQ fnoQ&dkIkbu (direction cosines or dc's) dgykrh gA

* For various activities in three dimensional geometry, one may refer to the Book *"A Hand Book for designing Mathematics Laboratory in Schools"*, NCERT, 2005

;fn ge L dh fn'kk foijhr dj nsrs g rks fnoQ&dk.k] viu lijdk e vFkkr $\pi-\alpha$, $\pi-\beta$ vkj $\pi-\gamma$ ls cny tkr gA bl idkj] fnoQ&dkslkbu oQ fpÉ cny tkr gA



vkoQfr 11-1

è;ku nhft,] vrfj{k e nh xb j[kk dk nks foijhr fn'kvk e c<k ldr g vkj blfy, blo fnoQ&dkslkbu oQ nk leg gA blfy, vrfj{k e Kkr j[kk oQ fy, fnoQ&dkslkbu oQ vf}rh; leg oQ fy,] ge Kkr j[kk dk ,d lfn'k j[kk yuk pkfg,A bu vf}rh; fnoQ&dkslkbu dk l, m vkj n oQ }kjk fufn"V fd, tkr gA

fVIi.kh vrfj{k e nh xb j[kk ;fn ey ¯cn l ugh xtjrh g rks bldh fnoQ&dkslkbu dk Kkr dju oQ fy,] ge ey ¯cn l nh xb j[kk oQ lekrj ,d j[kk [khpr gA vc ey ¯cn l bue l ,d lfn'k j[kk oQ fnoQ&vuikr Kkr djr g D;kfd nk lekrj j[kkvk oQ fnoQ&vuikrk oQ leg leku (ogh) gkr gA

,d j[kk oQ fnoQ&dkslkbu oQ lekuikrh l[;kvk dk j[kk oQ fnoQ&vuikr (direction ratios or *dr's*) dgr gA ;fn ,d j[kk oQ fnoQ&dkslkbu l, m, n o fnoQ&vuikr a, b, c gk rc fdlh 'kU;rj $\lambda \in \mathbf{R}$ oQ fy, $a = \lambda l$, $b=\lambda m$ vkj $c = \lambda n$

fVIi.kh oQN y[kd fnoQ&vuikrk dk fnoQ&l[;k, Hkh dgr gA

eku yhft, ,d j[kk oQ fnoQ&vuikr a, b, c vkj j[kk dh fnoQ&dkslkbu l, m, n gA rc

$$\frac{l}{a} = \frac{m}{b} = \frac{n}{c} = k \text{ (eku yhft,), } k \text{ ,d vpj gA}$$

blfy, $l = ak, m = bk, n = ck$ ... (1)

ijr $l^2 + m^2 + n^2 = 1$

blfy, $k^2 (a^2 + b^2 + c^2) = 1$

;k $k = \pm \dfrac{1}{\sqrt{a^2+b^2+c^2}}$

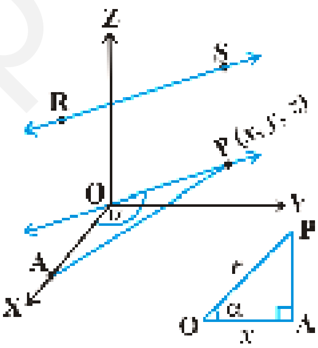
vr% (1) ls] js[kk dh fnd~&dkslkbu (*d.c.*'s)

$$l=\pm\frac{a}{\sqrt{a^2+b^2+c^2}},m=\pm\frac{b}{\sqrt{a^2+b^2+c^2}},n=\pm\frac{c}{\sqrt{a^2+b^2+c^2}}$$

fdlh js[kk osQ fy, ;fn js[kk osQ fnd~&vuqikr Øe'k% a, b, c g¨] rks $ka, kb, kc; k \neq 0$ Hkh fnd~&vuqikrksa dk ,d leqPp; gSA blfy, ,d js[kk osQ fnd~&vuqikrksa osQ nks leqPp; Hkh lekuqikrh gksxsaA vr% fdlh ,d js[kk osQ fnd~&vuqikrksa osQ vla[; leqPp; gksrs gSaA

11.2.1 js[kk dh fnd~&dkslkbu esa laca/ *(Relation between the direction cosines of a line)*

eku yhft, fd ,d js[kk RS dh fnd~&dkslkbu l, m, n gSaA ewy fcanq ls nh xbZ js[kk osQ lekarj ,d js[kk [khafp, vkSj bl ij ,d fcanq $P(x, y, z)$ yhft,A P ls x-v{k ij yac PA [khafp, (vkoQfr 11-2)A



vkoQfr 11-2

;fn OP = r. rks $\cos\alpha = \dfrac{OA}{OP} = \dfrac{x}{r}$. ftlls $x = lr$ izkIr gksrk gSA

blh izdkj $y = mr$ vkSj $z = nr$.

blfy, $x^2 + y^2 + z^2 = r^2 (l^2 + m^2 + n^2)$

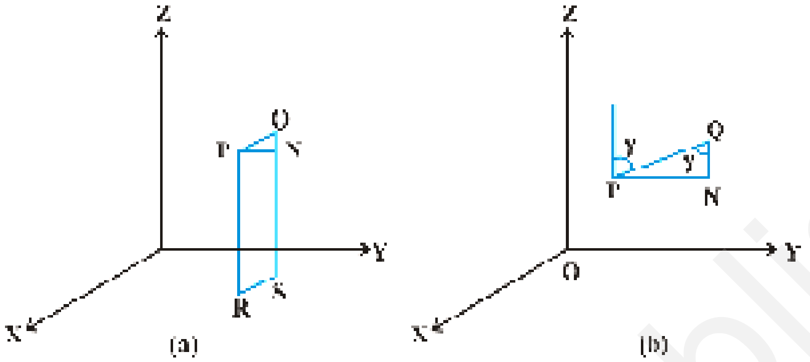
ijr $x^2 + y^2 + z^2 = r^2$

vr% $\mathbf{l^2 + m^2 + n^2 = 1}$

11.2.2 nks fcanqvksa dks feykus okyh js[kk dh fnd~&dkslkbu *(Direction cosines of a line passing through two points)*

D;ksafd nks fn, fcanqvksa ls gksdj tkus okyh js[kk vf}rh; gksrh gSA blfy, nks fn, x, fcanqvksa $P(x_1, y_1, z_1)$ vkSj $Q(x_2, y_2, z_2)$ ls xqtjus okyh js[kk dh fnd~&dkslkbu dks fuEu izdkj ls Kkr dj ldrs gSa (vkoQfr 11-3 (a))A

eku yhft, fd js[kk PQ dh fnd~&dkslkbu l, m, n gSa vkSj ;g x, y vkSj z-v{k osQ lkFk dks.k Øe'k% α, β, γ cukrh gSA eku yhft, P vkj Q ls yc [khfp, tks XY-ry dks R rFkk S ij feyrs gSaA P ls ,d vU; yc [khfp, tks QS dks N ij feyrk gSA vc ledks.k f=kHkqt PNQ esa, $\angle PQN = \gamma$ (vko`fr 11.3 (b)) blfy,



vko`fr 11-3

$$\cos\gamma = \frac{NQ}{PQ} = \frac{z_2 - z_1}{PQ}$$

blh izdkj $\cos\alpha = \dfrac{x_2 - x_1}{PQ}$ vkSj $\cos\beta = \dfrac{y_2 - y_1}{PQ}$

vr% ¯cnvksa $P(x_1, y_1, z_1)$ rFkk $Q(x_2, y_2, z_2)$ dks tksM+us okyh js[kk[kaM PQ fd fnd~&dkslkbu

$$\frac{x_2 - x_1}{PQ}, \frac{y_2 - y_1}{PQ}, \frac{z_2 - z_1}{PQ} \text{ gSa}$$

tgk¡ $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

fVIi.kh ¯cnvksa $P(x_1, y_1, z_1)$ rFkk $Q(x_2, y_2, z_2)$ dks tksM+us okyh js[kk[kaM oQs fnd~&vuqikr fuEu izdkj ls fy, tk ldrs gSa

x_2 ï x_1, y_2 ï y_1, z_2 ï z_1, ;k x_1 ï x_2, y_1 ï y_2, z_1 ï z_2

mnkgj.k 1 ;fn ,d js[kk x, y rFkk z-v{kksa dh /ukRed fn'kk oQs lkFk Øe'k% 90°, 60° rFkk 30° dk dks.k cukrh gS rks fnd~&dkslkbu Kkr dhft,A

gy eku yhft, js[kk dh fnd~&dkslkbu l, m, n gSaA rc $l = \cos 90° = 0$, $m = \cos 60° = \dfrac{1}{2}$, $n = \cos 30° = \dfrac{\sqrt{3}}{2}$

mnkgj.k 2 ;fn ,d js[kk oQ fnoQ&vuqikr 2] &1] &2 gSa rks blosQ fnoQ&dkslkbu Kkr dhft,A

gy fnoQ&dkslkbu fuEuor gSa

$$\frac{2}{\sqrt{2^2+(-1)^2+(-2)^2}},\ \frac{-1}{\sqrt{2^2+(-1)^2+(-2)^2}},\ \frac{-2}{\sqrt{2^2+(-1)^2+(-2)^2}}$$

vFkkZr $\frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}$

mnkgj.k 3 nks fcanqvksa (– 2, 4, – 5) vkSj (1, 2, 3) dks feykus okyh js[kk dh fnoQ&dkslkbu Kkr dhft,A

gy ge tkurs gSa fd nks fcanqvksa $P(x_1, y_1, z_1)$ vkSj $Q(x_2, y_2, z_2)$ dks feykus okyh js[kk dh fnoQ&dkslkbu

$$\frac{x_2-x_1}{PQ}, \frac{y_2-y_1}{PQ}, \frac{z_2-z_1}{PQ}$$

gSa tgk¡ $PQ = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

;gk¡ P vkSj Q Øe'k% (– 2, 4, – 5) vkSj (1, 2, 3) gSaA

blfy, $PQ = \sqrt{(1-(-2))^2+(2-4)^2+(3-(-5))^2} = \sqrt{77}$

blfy, nks fcanqvksa dks feykus okyh js[kk dh fnoQ&dkslkbu gSa%

$$\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}$$

mnkgj.k 4 x, y vkSj z-v{kksa dh fnoQ&dkslkbu Kkr dhft,A

gy x-v{k Øe'k% x, y vkSj z-v{k osQ lkFk 0°, 90° vkSj 90° osQ dks.k cukrk gSA blfy, x-v{k dh fnoQ&dkslkbu cos 0°, cos 90°, cos 90° vFkkZr 1,0,0 gSaA
blh izdkj y-v{k vkSj z-v{k dh fnoQ&dkslkbu Øe'k% 0, 1, 0 vkSj 0, 0, 1 gSaA

mnkgj.k 5 n'kkZb, fd fcanq A (2, 3, – 4), B (1, – 2, 3) vkSj C (3, 8, – 11) lajs[k gSaA

gy A vkSj B dks feykus okyh js[kk osQ fnoQ&vuqikr

1 – 2, – 2 – 3, 3 + 4 vFkkZr – 1, – 5, 7 gSaA

B vkSj C dks feykus okyh js[kk osQ fnoQ&vuqikr 3 – 1, 8 + 2, – 11 – 3, vFkkZr, 2, 10, – 14 gSaA

Li"V gS fd AB vkSj BC osQ fnoQ&vuqikr lekuqikrh gSaA vr% AB vkSj BC lekarj gSaA ijarq AB vkSj BC nksuksa esa B mHk;fu"B gSA vr% A, B, vkSj C lajs[k gSaA

i'ukoyh 11-1

1. ;fn ,d js[kk x, y vkSj z-v{kksa osQ lkFk Øe'k% 90°, 135°, 45° osQ dks.k cukrh gS rks blosQ fnd~&dkslkbu Kkr dhft,A
2. ,d js[kk dh fnd~&dkslkbu Kkr dhft, tks funsZ'kka{kksa osQ lkFk leku dks.k cukrh gSA
3. ;fn ,d js[kk osQ fnd~&vuqikr –18, 12, – 4, gSa rks blosQ fnd~&dkslkbu D;k gSa\
4. n'kkZb, fd ¯cnq (2, 3, 4), (– 1, – 2, 1), (5, 8, 7) lajs[k gSaA
5. ,d f=kHkqt dh Hkqtkvksa dh fnd~&dkslkbu Kkr dhft, ;fn f=kHkqt osQ 'kh"kZ ¯cnq (3, 5, – 4), (– 1, 1, 2) vkSj (– 5, – 5, – 2) gSaA

11.3 varfj{k esa js[kk dk lehdj.k (Equation of a Line in Space)

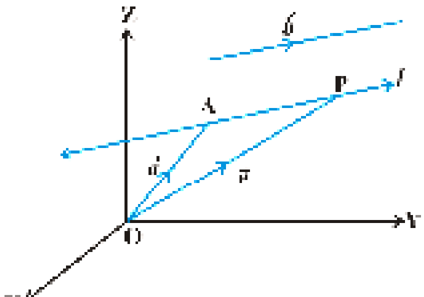
d{kk XI esa f}&foeh; ry esa js[kkvksa dk vè;;u djus osQ i'pkr vc ge varfj{k esa ,d js[kk osQ lfn'k rFkk dkrhZ; lehdj.kksa dks Kkr djsaxsA

,d js[kk vf}rh;r% fu/kZfjr gksrh gS] ;fn

(i) ;g fn, ¯cnq ls nh xbZ fn'kk ls gksdj tkrh gS] ;k
(ii) ;g nks fn, x, ¯cnqvksa ls gksdj tkrh gSA

11.3.1 fn, x, ¯cnq A ls tkus okyh rFkk fn, x, lfn'k $\vec{b}$ osQ lekarj js[kk dk lehdj.k *(Equation of a line through a given point A and parallel to a given vector $\vec{b}$)*

ledksf.kd funsZ'kk{k fudk; osQ ey ¯cnq O osQ lkis{k eku yhft, fd ¯cnq A dk lfn'k $\vec{a}$ gSA eku yhft, fd ¯cnq A ls tkus okyh rFkk fn, x, lfn'k $\vec{b}$ osQ lekarj js[kk l gSA eku yhft, fd l ij fLFkr fdlh LoSPN ¯cnq P dk fLFkfr lfn'k $\vec{r}$ gS (vkoQfr 11-4)A



vkoQfr 11-4

rc $\overrightarrow{AP}$ lfn'k $\vec{b}$ osQ lekarj gS vFkkZr $\overrightarrow{AP} = \lambda\vec{b}$, tgk¡ λ ,d okLrfod l[;k gSA

ijarq $\overrightarrow{AP} = \overrightarrow{OP} – \overrightarrow{OA}$

vFkkZr $\lambda\vec{b} = \vec{r} - \vec{a}$

foykser% ikpy λ osQ iR;sd eku osQ fy, ;g lehdj.k js[kk osQ fdlh ¯cnq P dh fLFkfr inku djrk gSA vr% js[kk dk lfn'k lehdj.k gS%

$$\vec{r} = \vec{a} + \lambda\vec{b} \qquad \ldots (1)$$

fVIi.kh ;fn $\vec{b} = a\hat{i} + b\hat{j} + c\hat{k}$ g rk j[kk oQ fnoQ&vuikr a, b, c g vkj foykerk ;fn ,d j[kk oQ fnoQ&vuikr a, b, c gk rk $\vec{b} = a\hat{i} + b\hat{j} + c\hat{k}$ j[kk oQ lekrj gkxkA ;gk b dk $|\vec{b}|$ u le>k tk,A

lfn'k :i l dkrh; :i 0;Rillu djuk (Derivation of Cartesian Form from Vector Form)

eku yhft, fd fn, ¯cn A oQ funs'kkd (x_1, y_1, z_1) g vkj j[kk dh fnoQ&dkslkbu a, b, c g eku yhft, fdlh ¯cn P oQ funs'kkd (x, y, z) gA rc

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k};\ \vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$

vkj

$$\vec{b} = a\hat{i} + b\hat{j} + c\hat{k}$$

bu ekuk dk (1) e ifrLFkfir djoQ $\hat{i}, \hat{j}$ vkj $\hat{k}$, oQ x.kkdk dh ryuk dju ij ge ikr g fd

$$x = x_1 + \lambda a;\ y = y_1 + \lambda b;\ z = z_1 + \lambda c \qquad \ldots (2)$$

; j[kk oQ ikpy lehdj.k gA (2) l ikpy λ dk foyksiu dju ij] ge ikr g%

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \qquad \ldots (3)$$

;g j[kk dk dkrh; lehdj.k gA

fVIi.kh ;fn j[kk dh fnoQ&dkslkbu l, m, n g] rk j[kk dk lehdj.k

$$\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$$ gA

mnkgj.k 6 ¯cn $(5, 2, -4)$ l tku okyh rFkk lfn'k $3\hat{i} + 2\hat{j} - 8\hat{k}$ oQ lekrj j[kk dk lfn'k rFkk dkrh; lehdj.kk dk Kkr dhft,A

gy ge Kkr g] fd

$$\vec{a} = 5\hat{i} + 2\hat{j} - 4\hat{k} \text{ vkj } \vec{b} = 3\hat{i} + 2\hat{j} - 8\hat{k}$$

blfy,] j[kk dk lfn'k lehdj.k g%

$$\vec{r} = 5\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(3\hat{i} + 2\hat{j} - 8\hat{k}) \quad [(1) \text{ l}]$$

pfd j[kk ij fLFkr fdlh ¯cn P(x, y, z) dh fLFkfr lfn'k $\vec{r}$ g] blfy,

$$x\hat{i} + y\hat{j} + z\hat{k} = 5\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(3\hat{i} + 2\hat{j} - 8\hat{k})$$

$$= (5 + 3\lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (-4 - 8\lambda)\hat{k}$$

λ dk foyksiu djus ij ge ikrs gSa fd

$$\frac{x-5}{3} = \frac{y-2}{2} = \frac{z+4}{-8}$$

tks js[kk osQ lehdj.k dk dkrhZ; :i gSA

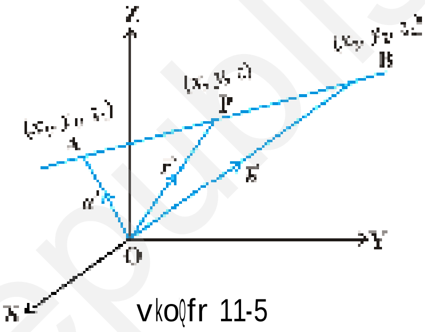
11.3.2 nks fn, x, fcanqvksa ls tkus okyh js[kk dk lehdj.k *(Equation of a line passing through two given points)*

eku yhft, ,d js[kk ij fLFkr nks fcanqvksa $A(x_1, y_1, z_1)$ vkSj $B(x_2, y_2, z_2)$, osQ fLFkfr lfn'k Øe'k% $\vec{a}$ vkSj $\vec{b}$ gSa (vkÑfr 11.5)A

eku yhft, $\vec{r}$,d LosPN fcanq P dk fLFkfr lfn'k gSA rc P js[kk ij gS ;fn vkSj osQoy ;fn $\overrightarrow{AP} = \vec{r} - \vec{a}$ rFkk $\overrightarrow{AB} = \vec{b} - \vec{a}$ lajs[k lfn'k gSa blfy, P js[kk ij fLFkr gS ;fn vkSj osQoy ;fn

$$\vec{r} - \vec{a} = \lambda(\vec{b} - \vec{a})$$

;k $$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a}),\ \lambda \in \mathbf{R} \quad \ldots (1)$$



vkÑfr 11-5

tks js[kk dk lfn'k lehdj.k gSA

lfn'k :i ls dkrhZ; :i O;qRiUu djuk

ge ikrs gSa fd

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k},\ \vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}, \text{ vkSj } \vec{b} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$$

bu ekuksa dks (1) esa ifrLFkkfir djus ij ge ikrs gSa fd

$$x\hat{i} + y\hat{j} + z\hat{k} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k} + \lambda[(x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}]$$

$\hat{i}, \hat{j}, \hat{k}$ osQ x.kkadksa dh rqyuk djus ij ge ikrs gSa fd

$$x = x_1 + \lambda(x_2 - x_1);\ y = y_1 + \lambda(y_2 - y_1);\ z = z_1 + \lambda(z_2 - z_1)$$

λ dk foyksiu djus ij ge ikrs gSa fd

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

tks js[kk osQ lehdj.k dk dkrhZ; :i gSA

mnkgj.k 7 fcanqvksa $(-1, 0, 2)$ vkSj $(3, 4, 6)$ ls gksdj tkus okyh js[kk dk lfn'k lehdj.k Kkr dhft,A

gy eku yhft, $\vec{a}$ vkSj $\vec{b}$ fcanqvksa $A(-1, 0, 2)$ vkSj $B(3, 4, 6)$ osQ fLFkfr lfn'k gSaA rc $\vec{a} = -\hat{i} + 2\hat{k}$

vkj $\vec{b} = 3\hat{i} + 4\hat{j} + 6\hat{k}$

blfy, $\vec{b} - \vec{a} = 4\hat{i} + 4\hat{j} + 4\hat{k}$

eku yhft, fd j[kk ij fLFkr fdlh LoPN ¯cnq P dk fLFkfr lfn'k $\vec{r}$ gA vr% j[kk dk lfn'k lehdj.k

$$\vec{r} = -\hat{i} + 2\hat{k} + \lambda(4\hat{i} + 4\hat{j} + 4\hat{k})$$

mnkgj.k **8** ,d j[kk dk dkrh; lehdj.k $\frac{x+3}{2} = \frac{y-5}{4} = \frac{z+6}{2}$ gA bl j[kk dk lfn'k lehdj.k Kkr dhft,A

gy fn, x, lehdj.k dk ekud :i

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

l ryuk dju ij ge ikr g fd $x_1 = -3, y_1 = 5, z_1 = -6; a = 2, b = 4, c = 2$

bl idkj vHkh"V j[kk ¯cn (– 3, 5, – 6) l gkdj tkrh g rFkk lfn'k $2\hat{i} + 4\hat{j} + 2\hat{k}$ o lekrj gA eku yhft, fd j[kk ij fLFkr fdlh ¯cn dh fLFkfr lfn'k $\vec{r}$ g rk j[kk dk lfn'k lehdj.k

$$\vec{r} = (-3\hat{i} + 5\hat{j} - 6\hat{k}) + \lambda\,(2\hat{i} + 4\hat{j} + 2\hat{k})$$

}kjk inÙk gA

11.4 nk j[kkvk o eè; dk.k (Angle between two lines)

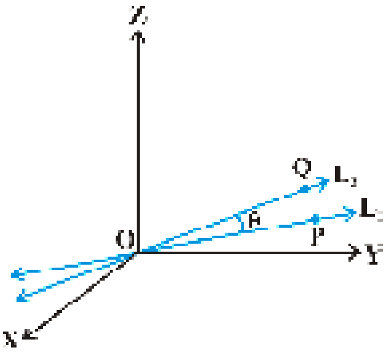
eku yhft, fd L_1 vkj L_2 ey ¯cn l xtju okyh nk j[kk, g ftuo fno&vuikr Øe'k% a_1, b_1, c_1 vkj a_2, b_2, c_2, gA iu% eku yhft,fd L_1 ij ,d ¯cn P rFkk L_2 ij ,d ¯cn Q gA vko`fr 11-6 e fn, x, lfn'k OP vkj OQ ij fopkj dhft,A eku yhft, fd OP vkj OQ o chp U;u dk.k θ gA vc Lej.k dhft, fd lfn'kk OP vkj OQ o ?kVd Øe'k% a_1, b_1, c_1 vkj a_2, b_2, c_2 gA blfy, muo chp dk dk.k θ

$$\cos\theta = \left|\frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\,\sqrt{a_2^2+b_2^2+c_2^2}}\right|$$ }kjk inÙk gA

iu% sin θ o :i e] j[kkvk o chp dk dk.k

$$\sin\theta = \sqrt{1-\cos^2\theta}$$ l inÙk g

$$= \sqrt{1 - \frac{(a_1a_2 + b_1b_2 + c_1c_2)^2}{\left(a_1^2+b_1^2+c_1^2\right)\left(a_2^2+b_2^2+c_2^2\right)}}$$



vko`fr 11-6

$$= \frac{\sqrt{(a_1^2+b_1^2+c_1^2)(a_2^2+b_2^2+c_2^2)-(a_1a_2+b_1b_2+c_1c_2)^2}}{\sqrt{(a_1^2+b_1^2+c_1^2)}\sqrt{(a_2^2+b_2^2+c_2^2)}}$$

$$= \frac{\sqrt{(a_1\ b_2 - a_2\ b_1)^2 + (b_1\ c_2 - b_2\ c_1)^2 + (c_1\ a_2 - c_2\ a_1)^2}}{\sqrt{(a_1^2 + b_1^2 + c_1^2)}\ \sqrt{a_2^2 + b_2^2 + c_2^2}} \qquad \ldots (2)$$

fVIi.kh mlI fLFkfr e tc j[kk,¡ L$_1$ vkj L$_2$ ey ¯cn l ugh xtjrh g rk ge L$_1$ vkj L$_2$ o¶ lekrj ey ¯cn l xtju okyh j[kk, Øe'k% L$'_1$ o L$'_2$ yr gA ;fn j[kkvk L$_1$ vkj L$_2$ o¶ fno¶&vuikrk o¶ ctk; fno¶&dkl kbu nh xb gk tl L$_1$ o¶ fy, l_1, m_1, n_1 vkj L$_2$ o¶ fy, l_2, m_2, n_2 rk (1) vkj (2) fuEufyf[kr ik:i yxA

$$\cos\theta = |l_1 l_2 + m_1 m_2 + n_1 n_2| \ (\text{D;kfd } l_1^2+m_1^2+n_1^2=1 = l_2^2+m_2^2+n_2^2) \qquad \ldots (3)$$

vkj
$$\sin\theta = \sqrt{(l_1 m_2 - l_2 m_1)^2 - (m_1 n_2 - m_2 n_1)^2 + (n_1 l_2 - n_2 l_1)^2} \qquad \ldots (4)$$

fno¶&vuikr a_1, b_1, c_1 vkj a_2, b_2, c_2 okyh j[kk,

(i) ycor g] ;fn $\theta = 90°$, vFkkr (1) l $\boldsymbol{a_1a_2 + b_1b_2 + c_1c_2 = 0}$

(ii) lekrj g] ;fn $\theta = 0$, vFkkr (2) l $\boldsymbol{\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}}$

vc ge nk j[kkvk o¶ chp dk dk.k Kkr djx ftuo¶ lehdj.k fn, x, gA ;fn mu j[kkvk $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ vkj $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ o¶ chp U;u dk.k θ g

rc
$$\cos\theta = \left|\frac{\vec{b}_1\cdot\vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right|$$

dkrh; :i e ;fn j[kkvk%
$$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1} \qquad \ldots (1)$$

vkj
$$\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2} \qquad \ldots (2)$$

o¶ chp dk dk.k θ g tgk j[kk, (1) o (2) o¶ fno¶&vuikr Øe'k% a_1, b_1, c_1 rFkk a_2, b_2, c_2 g rc

$$\cos\theta = \left|\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}\right|$$

mnkgj.k 9 fn, x, j[kk&;Xe

$$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$$

vkj $\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$

oQ eè; dk.k Kkr dhft,

gy eku yhft, $\vec{b}_1 = \hat{i} + 2\hat{j} + 2\hat{k}$ vkj $\vec{b}_2 = 3\hat{i} + 2\hat{j} + 6\hat{k}$

nkuk j[kkvk oQ eè; dk.k θ g] blfy,

$$\cos\theta = \left|\frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right| = \left|\frac{(\hat{i} + 2\hat{j} + 2\hat{k}) \cdot (3\hat{i} + 2\hat{j} + 6\hat{k})}{\sqrt{1+4+4}\sqrt{9+4+36}}\right|$$

$$= \left|\frac{3+4+12}{3\times 7}\right| = \frac{19}{21}$$

vr% $\theta = \cos^{-1}\left(\frac{19}{21}\right)$

mnkgj.k 10 j[kk&;Xe%

$$\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$$

vkj $$\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-5}{2}$$

oQ eè; dk.k Kkr dhft,A

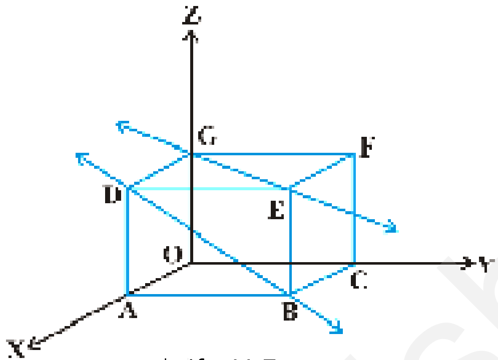
gy igyh j[kk oQ fnoQ&vuikr 3] 5] 4 vkj nljh j[kk oQ fnoQ&vuikr 1] 1] 2 gA ;fn muoQ chp dk dk.k θ gk rc

$$\cos\theta = \left|\frac{3.1+5.1+4.2}{\sqrt{3^2+5^2+4^2}\sqrt{1^2+1^2+2^2}}\right| = \frac{16}{\sqrt{50}\sqrt{6}} = \frac{16}{5\sqrt{2}\sqrt{6}} = \frac{8\sqrt{3}}{15}$$

vr% vHkh"V dk.k $\cos^{-1}\left(\frac{8\sqrt{3}}{15}\right)$ gA

11.5 nk j[kkvk oQ eè; U;ure njh (Shortest Distance between two lines)

vrfj{k e ;fn nk j[kk, ijLij ifrPNn djrh g rk muoQ chp dh U;ure njh 'kU; gA vkj vrfj{k e ;fn nk j[kk, lekrj g rk muoQ chp dh U;ure njh] muoQ chp ycor njh gkxh vFkkr ,d j[kk oQ ,d ¯cn l nljh j[kk ij [khpk x;k ycA



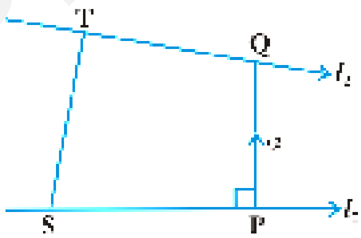
vkoQfr 11-7

blosQ vfrfjDr vrfj{k esa ,slh Hkh js[kk,¡ gksrh gSa tks u rks ifrPNsnh vkSj u gh lekarj gksrh gSaA okLro esa ,slh js[kkvksa osQ ;qXe vlery; gksrs gSa vkSj bUgsa fo"ker; js[kk,¡ (skew lines) dgrs gSaA mnkgj.kr;k ge vko`Qfr 11-7 esa x, y vkSj z-v{kksa osQ vuqfn'k Øe'k% 1] 3] 2 bdkbZ osQ vkdkj okys dejs ij fopkj djrs gSaA

js[kk GE Nr osQ fod.kZ osQ vuqfn'k gS vkSj js[kk DB, A osQ Bhd Åij Nr osQ dksus ls xqtjrh gqbZ nhokj osQ fod.kZ osQ vuqfn'k gSA ;s js[kk,¡ fo"ker; gSa D;ksafd os lekarj ugha gSa vkSj dHkh feyrh Hkh ugha gSaA

nks js[kkvksa osQ chp U;wure nwjh ls gekjk vfHkizk; ,d ,slh js[kk[kaM ls gS tks ,d js[kk ij fLFkr ,d ¯cnq dks nwljh js[kk ij fLFkr vU; ¯cnq dks feykus ls izkIr gks rkfd bldh yackbZ U;wure gksA U;wure nwjh js[kk[kaM nksuks fo"ker; js[kkvksa ij yac gksxkA

11.5.1 nks fo"ker; js[kkvksa osQ chp dh nwjh (*Distance between two skew lines*)



vko`Qfr 11-8

vc ge js[kkvksa osQ chp dh U;wure nwjh fuEufyf[kr fof/ ls Kkr djrs gSaA eku yhft, l_1 vkSj l_2 nks fo"ker; js[kk,¡ gSa ftuosQ lehdj.k (vko`Qfr 11.8) fuEufyf[kr gSa%

$$\vec{r} = \vec{a}_1 + \lambda \vec{b}_1 \qquad \text{... (1)}$$

vkSj
$$\vec{r} = \vec{a}_2 + \mu \vec{b}_2 \qquad \text{... (2)}$$

js[kk l_1 ij dksbZ ¯cnq S ftldh fLFkfr lfn'k $\vec{a}_1$ vkSj l_2 ij dksbZ ¯cnq T ftldh fLFkfr lfn'k $\vec{a}_2$ gS] yhft,A rc U;wure nwjh lfn'k dk ifjek.k] ST dk U;wure nwjh dh fn'kk esa iz{ksi dh eki osQ leku gksxk (vuqPNsn 10.6.2)A

;fn l_1 vkSj l_2 osQ chp dh U;wure nwjh lfn'k $\overrightarrow{PQ}$ gS rks ;g nksuksa $\vec{b}_1$ vkSj $\vec{b}_2$ ij yac gksxhA $\overrightarrow{PQ}$ dh fn'kk esa bdkbZ lfn'k $\hat{n}$ blfy, idkj gksxh fd

$$\hat{n} = \frac{\vec{b}_1 \times \vec{b}_2}{|\vec{b}_1 \times \vec{b}_2|} \qquad \text{... (3)}$$

rc $$\overrightarrow{PQ} = d\,\hat{n}$$

tgk d, U;ure njh lfn'k dk ifjek.k gA eku yhft, $\overrightarrow{ST}$ vkj $\overrightarrow{PQ}$ oQ chp dk dk.k θ g] rc

$$PQ = ST\,|\cos\theta|$$

ijr $$\cos\theta = \left|\frac{\overrightarrow{PQ}\cdot\overrightarrow{ST}}{|\overrightarrow{PQ}|\,|\overrightarrow{ST}|}\right|$$

$$= \left|\frac{d\,\hat{n}\cdot(\vec{a}_2 - \vec{a}_1)}{d\ ST}\right| \quad (\text{D;kfd } \overrightarrow{ST} = \vec{a}_2 - \vec{a}_1)$$

$$= \left|\frac{(\vec{b}_1 \times \vec{b}_2)\cdot(\vec{a}_2 - \vec{a}_1)}{ST\,\left|\vec{b}_1 \times \vec{b}_2\right|}\right| \quad ((3)\text{ oQ }\}\text{kjk})$$

blfy, vHkh"V U;ure njh

$$d = PQ = ST\,|\cos\theta|$$

;k $$d = \left|\frac{(\vec{b}_1 \times \vec{b}_2)\cdot(\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|}\right| \text{ gA}$$

dkrh; :i **(Cartesian Form)**

j[kkvk%

$$l_1 : \frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$$

vkj $$l_2 : \frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}$$

oQ chp dh U;ure njh g%

$$\frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2 + (a_1b_2 - a_2b_1)^2}}$$

11.5.2 lekrj j[kkvksa osQ chp dh njh *(Distance between parallel lines)*

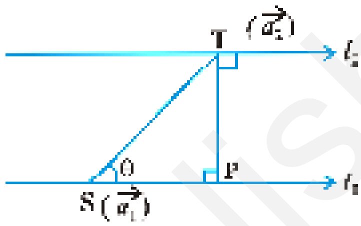
;fn nks j[kk,¡ l_1 ;fn l_2 lekrj gSa rks os leryh; gksrh gSaA ekuk nh xbZ j[kk,¡ Øe'k%

$$\vec{r} = \vec{a}_1 + \lambda \vec{b} \qquad \ldots (1)$$

vkSj

$$\vec{r} = \vec{a}_2 + \mu \vec{b} \qquad \ldots (2)$$

gSa] tgk¡ l_1 ij fcanq S dk fLFkfr lfn'k $\vec{a}_1$ vkSj l_2 ij fcanq T dk fLFkfr lfn'k $\vec{a}_2$ gS (vkoQfr 11.9)



vkoQfr 11-9

D;ksafd l_1, vkSj l_2 leryh; gSaA ;fn fcanq T ls l_1 ij Mkys x, yac dk ikn P gS rc j[kkvksa l_1 vkSj l_2 osQ chp dh njh = |TP|

eku yhft, fd lfn'kksa $\overrightarrow{ST}$ vkSj $\vec{b}$ osQ chp dk dks.k θ gSA rc]

$$\vec{b} \times \overrightarrow{ST} = (|\vec{b}|\,|\overrightarrow{ST}| \sin \theta)\,\hat{n} \qquad \ldots (3)$$

tgk¡ j[kkvksa l_1 vkSj l_2 osQ ry ij yac bdkbZ lfn'k $\hat{n}$ gSA

ijarq

$$\overrightarrow{ST} = \vec{a}_2 - \vec{a}_1$$

blfy, (3) ls ge izkIr djrs gSa fd

$$\vec{b} \times (\vec{a}_2 - \vec{a}_1) = |\vec{b}|\,\mathrm{PT}\,\hat{n} \qquad (\text{D;ksafd } \mathrm{PT} = \mathrm{ST} \sin \theta)$$

vFkkZr~

$$|\vec{b} \times (\vec{a}_2 - \vec{a}_1)| = |\vec{b}|\,\mathrm{PT} \cdot 1 \qquad (\text{as } |\hat{n}| = 1)$$

blfy, Kkr j[kkvksa osQ chp U;wure njh

$$d = |\overrightarrow{\mathrm{PT}}| = \left| \frac{\vec{b} \times (\vec{a}_2 - \vec{a}_1)}{|\vec{b}|} \right| \text{ gSA}$$

mnkgj.k 11 j[kkvksa l_1 vkSj l_2 osQ chp dh U;wure njh Kkr dhft, ftuosQ lfn'k lehdj.k gSa%

$$\vec{r} = \hat{i} + \hat{j} + \lambda\,(2\hat{i} - \hat{j} + \hat{k}) \qquad \ldots (1)$$

vkSj

$$\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu\,(3\hat{i} - 5\hat{j} + 2\hat{k}) \qquad \ldots (2)$$

gy lehdj.k (1) o (2) dh $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ vkSj $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$, ls rqyuk djus ij ge izkIr djrs gSa fd

$$\vec{a}_1 = \hat{i} + \hat{j},\quad \vec{b}_1 = 2\hat{i} - \hat{j} + \hat{k}$$

$$\vec{a}_2 = 2\hat{i} + \hat{j} - \hat{k} \text{ vkSj } \vec{b}_2 = 3\hat{i} - 5\hat{j} + 2\hat{k}$$

bIfy, $\vec{a}_2 - \vec{a}_1 = \hat{i} - \hat{k}$

vkj $\vec{b}_1 \times \vec{b}_2 = (2\hat{i} - \hat{j} + \hat{k}) \times (3\hat{i} - 5\hat{j} + 2\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix} = 3\hat{i} - \hat{j} - 7\hat{k}$$

bl idkj $|\vec{b}_1 \times \vec{b}_2| = \sqrt{9+1+49} = \sqrt{59}$

bIfy, nh xb j[kkvk oQ chp dh U;ure njh

$$d = \left| \frac{(\vec{b}_1 \times \vec{b}_2)\cdot(\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right| = \frac{|3-0+7|}{\sqrt{59}} = \frac{10}{\sqrt{59}}$$

mnkgj.k **12** fuEufyf[kr nh xb j[kkvk l_1 vkj l_2 :

$$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

vkj $\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$ oQ chp U;ure njh Kkr dhft,A

gy nkuk j[kk, lekrj gA (D;k\) ge ikIr g fd

$$\vec{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k},\ \vec{a}_2 = 3\hat{i} + 3\hat{j} - 5\hat{k} \text{ vkj } \vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

bIfy, j[kkvk oQ chp dh njh

$$d = \left| \frac{\vec{b} \times (\vec{a}_2 - \vec{a}_1)}{|\vec{b}|} \right| = \left| \frac{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ 2 & 1 & -1 \end{vmatrix}}{\sqrt{4+9+36}} \right|$$

$$= \frac{|-9\hat{i} + 14\hat{j} - 4\hat{k}|}{\sqrt{49}} = \frac{\sqrt{293}}{\sqrt{49}} = \frac{\sqrt{293}}{7} \text{ gA}$$

i'ukoyh 11-2

1. n'kkb, fd fnoQ&dkIkbu $\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13};\ \frac{4}{13}, \frac{12}{13}, \frac{3}{13};\ \frac{3}{13}, \frac{-4}{13}, \frac{12}{13}$ okyh rhu j[kk, ijLij ycor gA
2. n'kkb, fd ¯cnvk $(1, -1, 2), (3, 4, -2)$ l gkdj tku okyh j[kk ¯cnvk $(0, 3, 2)$ vkj $(3, 5, 6)$ l tku okyh j[kk ij yc gA

3. n'kkb, fd ¯cnvk (4, 7, 8), (2, 3, 4) l gkdj tku okyh j[kk] ¯cnvk (ï 1, ï 2, 1), (1, 2, 5) l tku okyh j[kk oQ lekrj gA

4. ¯cn (1, 2, 3) l x $3\hat{i}+2\hat{j}-2\hat{k}$ oQ lekrj gA

5. ¯cn ftldh fLFkfr lfn'k $2\hat{i}-\hat{j}+4\hat{k}$ l x $\hat{i}+2\hat{j}-\hat{k}$ dh fn'kk e tku okyh j[kk dk lfn'k vkj dkrh; :ik e lehdj.k Kkr dhft,A

6. ml j[kk dk dkrh; lehdj.k Kkr dhft, tk ¯cn (ï 2, 4, ï 5) l tkrh g vkj $\frac{x+3}{3}=\frac{y-4}{5}=\frac{z+8}{6}$ oQ lekrj gA

7. ,d j[kk dk dkrh; lehdj.k $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$ gA bldk lfn'k lehdj.k Kkr dhft,A

8. ey ¯cn vkj (5, ï 2, 3) l tku okyh j[kk dk lfn'k rFkk dkrh; :ik e lehdj.k Kkr dhft,A

9. ¯cnvk (3, ï 2, ï 5), vkj (3, ï 2, 6) l x e lehdj.k dk Kkr dhft,A

10. fuEufyf[kr j[kk&;Xek oQ chp dk dk.k Kkr dhft,%

(i) $\vec{r}=2\hat{i}-5\hat{j}+\hat{k}+\lambda(3\hat{i}+2\hat{j}+6\hat{k})$ vkj

$\vec{r}=7\hat{i}-6\hat{k}+\mu(\hat{i}+2\hat{j}+2\hat{k})$

(ii) $\vec{r}=3\hat{i}+\hat{j}-2\hat{k}+\lambda(\hat{i}-\hat{j}-2\hat{k})$ vkj

$\vec{r}=2\hat{i}-\hat{j}-56\hat{k}+\mu(3\hat{i}-5\hat{j}-4\hat{k})$

11. fuEufyf[kr j[kk&;Xek oQ chp dk dk.k Kkr dhft,%

(i) $\frac{x-2}{2}=\frac{y-1}{5}=\frac{z+3}{-3}$ vkj $\frac{x+2}{-1}=\frac{y-4}{8}=\frac{z-5}{4}$

(ii) $\frac{x}{2}=\frac{y}{2}=\frac{z}{1}$ vkj $\frac{x-5}{4}=\frac{y-2}{1}=\frac{z-3}{8}$

12. p dk eku Kkr dhft, rkfd j[kk, $\frac{1-x}{3}=\frac{7y-14}{2p}=\frac{z-3}{2}$

vkj $\frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$ ijLij yc gkA

13. fn[kkb, fd j[kk,¡ $\frac{x-5}{7}=\frac{y+2}{-5}=\frac{z}{1}$ vkSj $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ ijLij yac gSaA

14. j[kkvksa $\vec{r}=(\hat{i}+2\hat{j}+\hat{k})+\lambda(\hat{i}-\hat{j}+\hat{k})$ vkSj $\vec{r}=2\hat{i}-\hat{j}-\hat{k}+\mu(2\hat{i}+\hat{j}+2\hat{k})$ oQ chp dh U;wure njh Kkr dhft,%

15. j[kkvksa $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$ vkSj $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$ oQ chp dh U;wure njh Kkr dhft,A

16. j[kk,¡ ftuoQ lfn'k lehdj.k fuEufyf[kr gSa] oQ chp dh U;wure njh Kkr dhft,%

$\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(\hat{i}-3\hat{j}+2\hat{k})$ vkSj $\vec{r}=4\hat{i}+5\hat{j}+6\hat{k}+\mu(2\hat{i}+3\hat{j}+\hat{k})$

17. j[kk,¡ ftudh lfn'k lehdj.k fuEufyf[kr gSa] oQ chp dh U;wure Kkr dhft,%

$\vec{r}=(1-t)\hat{i}+(t-2)\hat{j}+(3-2t)\hat{k}$ vkSj $\vec{r}=(s+1)\hat{i}+(2s-1)\hat{j}-(2s+1)\hat{k}$

11.6 lery (Plane)

,d lery dk vf}rh; :i ls Kkr fd;k tk ldrk gS ;fn fuEufyf[kr esa ls dksbZ ,d 'krZ Kkr gks%

(i) lery dk vfHkyEc vkSj ewy ¯cnq ls lery dh njh Kkr gS] vFkkZr vfHkyEc :i esa lery dk lehdj.kA

(ii) ;g ,d ¯cnq ls xqtjrk gS

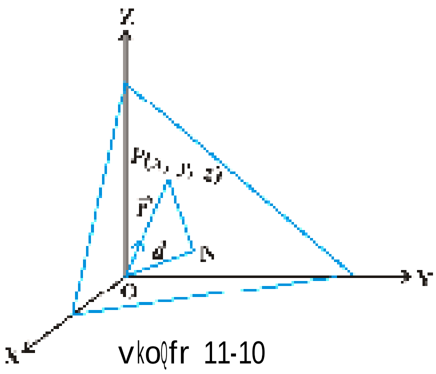
(iii) ;g fn, x, rhu vlaj[k ¯cnqvksa ls xqtjrk gS

vc ge leryksa oQ lfn'k vkSj dkrhZ; lehdj.kksa dks izkIr djsaxsA

11.6.1 vfHkyEc :i esa lery dk lehdj.k *(Equation of a Plane in normal form)*

,d lery ij fopkj dhft, ftldh ewy ¯cnq ls ycor njh $d\ (d \neq 0)$ gS (vkoQfr 11-10)A

;fn $\overrightarrow{ON}$ ewy ¯cnq ls ry ij yac gS rFkk $\overrightarrow{ON}$ oQ vuqfn'k $\hat{n}$ ek=kd vfHkyEc lfn'k gS rc $\overrightarrow{ON}=d\,\hat{n}$ gSA eku yhft, fd lery ij dksbZ ¯cnq P gSA blfy,] $\overrightarrow{NP}$, $\overrightarrow{ON}$ ij yac gSA



vkoQfr 11-10

vr% $\overrightarrow{NP}\cdot\overrightarrow{ON}=0$ ... (1)

eku yhft, P dh fLFkfr lfn'k $\vec{r}$ gS rks $\overrightarrow{NP}=\vec{r}-d\,\hat{n}$ (D;ksafd $\overrightarrow{ON}+\overrightarrow{NP}=\overrightarrow{OP}$)

bl idkj (1) dk :i fuEufyf[kr g%

$$(\vec{r} - d\,\hat{n})\cdot d\,\hat{n} = 0$$

;k $$(\vec{r} - d\,\hat{n})\cdot \hat{n} = 0 \ (d \neq 0)$$

;k $$\vec{r}\cdot\hat{n} - d\,\hat{n}\cdot\hat{n} = 0$$

vFkkr $$\vec{r}\cdot\hat{n} = d \quad (\text{D;kfd } \hat{n}\cdot\hat{n} = 1)$$ é (2)

;g lery dk lfn'k lehdj.k gA

dkrh; :i **(Cartesian Form)**

lery dk lfn'k lehdj.k g tgk $\hat{n}$ lery o vfHkyc bdkb lfn'k gA eku yhft, lery ij dkb cn P(x, y, z) gA rc

$$\overrightarrow{OP} = \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

eku yhft, $\hat{n}$ dh fno&dkslkbu l, m, n gA rc

$$\hat{n} = l\hat{i} + m\hat{j} + n\hat{k}$$

$\vec{r}\cdot\hat{n}$ o eku dk (2) e ifrLFkkfir dju ij ge ikr g]

$$(x\hat{i} + y\hat{j} + z\hat{k})\cdot(l\hat{i} + m\hat{j} + n\hat{k}) = d$$

vFkkr $$\boldsymbol{lx + my + nz = d}$$ ... (3)

;g lery dk dkrh; lehdj.k gA

fVli.kh lehdj.k (3) inf'kr djrk g fd ;fn $\vec{r}\cdot(a\hat{i} + b\hat{j} + c\hat{k}) = d$,d lery dk lfn'k lehdj.k g rk $ax + by + cz = d$ lery dk dkrh; lehdj.k g tgk a, b vkj c lery o vfHkyc o fno&vuikr gA

mnkgj.k 13 ml lery dk lfn'k lehdj.k Kkr dhft, tk ey cn l $\frac{6}{\sqrt{29}}$ dh njh ij g vkj ey cn l bldk vfHkyc lfn'k $2\hat{i} - 3\hat{j} + 4\hat{k}$ gA

gy eku yhft, $\vec{n} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ gA rc

$$\hat{n} = \frac{\vec{n}}{|\vec{n}|} = \frac{2\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{4 + 9 + 16}} = \frac{2\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{29}}$$

blfy, lery dk vHkh"V lehdj.k

$$\vec{r}\cdot\left(\frac{2}{\sqrt{29}}\hat{i}+\frac{-3}{\sqrt{29}}\hat{j}+\frac{4}{\sqrt{29}}\hat{k}\right)=\frac{6}{\sqrt{29}}$$ gA

mnkgj.k **14** lery $\vec{r}\cdot(6\hat{i}-3\hat{j}-2\hat{k})+1=0$ ij ey ¯cn l Mky x, yc bdkb lfn'k dh fnoQ&dklkbu Kkr dhft,A

gy lery oQ Kkr lehdj.k dk bl idkj 0;Dr fd;k tk ldrk g%

$$\vec{r}\cdot(-6\hat{i}+3\hat{j}+2\hat{k})=1 \qquad \text{... (1)}$$

vc $$|-6\hat{i}+3\hat{j}+2\hat{k}|=\sqrt{36+9+4}=7$$

blfy, (1) oQ nkuk i{kk dk 7 l Hkkx dju ij ge ikr g fd

$$\vec{r}\cdot\left(-\frac{6}{7}\hat{i}+\frac{3}{7}\hat{j}+\frac{2}{7}\hat{k}\right)=\frac{1}{7}$$

tk fd lery dk lehdj.k $\vec{r}\cdot\hat{n}=d$ oQ :i dk gA

bl Li"V g fd $\hat{n}=-\frac{6}{7}\hat{i}+\frac{3}{7}\hat{j}+\frac{2}{7}\hat{k}$ lery oQ yc bdkb lfn'k g tk ey ¯cn l xtjrk gA bl idkj $\hat{n}$ dh fnoQ&dklkbu $\frac{-6}{7},\frac{3}{7},\frac{2}{7}$ gA

mnkgj.k **15** lery $2x-3y+4z-6=0$ dh ey ¯cn l njh Kkr dhft,A

gy D;kfd ry oQ vfHkyc oQ fnoQ&vuikr 2, –3, 4 g blfy, bldh fnoQ&dklkbu g%

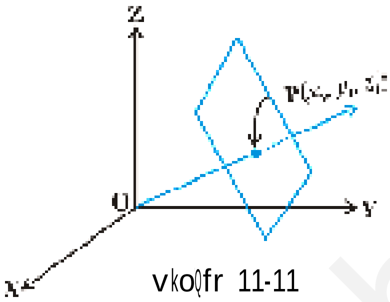
$$\frac{2}{\sqrt{2^2+(-3)^2+4^2}},\frac{-3}{\sqrt{2^2+(-3)^2+4^2}},\frac{4}{\sqrt{2^2+(-3)^2+4^2}}$$, vFkkr $\frac{2}{\sqrt{29}},\frac{-3}{\sqrt{29}},\frac{4}{\sqrt{29}}$

blfy, lehdj.k $2x-3y+4z-6=0$ vFkkr $2x-3y+4z=6$ dk $\sqrt{29}$ l Hkkx dju ij ge ikIr djr g%

$$\frac{2}{\sqrt{29}}x+\frac{-3}{\sqrt{29}}y+\frac{4}{\sqrt{29}}z=\frac{6}{\sqrt{29}}$$

vkj ;g $lx+my+nz=d$, oQ :i e g tgk ey ¯cn l lery dh njh d gA blfy, lery dh ey ¯cn l njh $\frac{6}{\sqrt{29}}$ gA

mnkgj.k 16 ey ¯cn l lery $2x - 3y + 4z - 6 = 0$ ij Mky x, yc oQ ikn oQ fun'kkd Kkr dhft,A



vkoQfr 11-11

gy ekuk yhft, ey ¯cn l lery ij Mky x, yc oQ ikn P oQ fun'kkd (x_1, y_1, z_1) g (vkoQfr 11.11)A

rc j[kk OP oQ fnoQ&vuikr x_1, y_1, z_1 gA

lery dh lehdj.k dk vfHkyc oQ :i e fy[ku ij ge ikr g fd

$$\frac{2}{\sqrt{29}}x - \frac{3}{\sqrt{29}}y + \frac{4}{\sqrt{29}}z = \frac{6}{\sqrt{29}}$$

tgk OP oQ fnoQ&vuikr $\frac{2}{\sqrt{29}}, \frac{-3}{\sqrt{29}}, \frac{4}{\sqrt{29}}$ gA

D;kfd ,d j[kk oQ fnoQ&dkl kbu vkj fnoQ&vuikr lekuikrh gkr gA vr%

$$\frac{x_1}{\frac{2}{\sqrt{29}}} = \frac{y_1}{\frac{-3}{\sqrt{29}}} = \frac{z_1}{\frac{4}{\sqrt{29}}} = k$$

vFkkr $x_1 = \frac{2k}{\sqrt{29}}, y_1 = \frac{-3k}{\sqrt{29}}, z_1 = \frac{4k}{\sqrt{29}}$

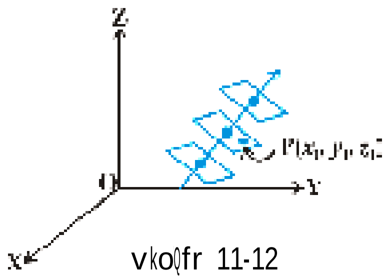
bu ekuk dk lery oQ lehdj.k e ifrLFkkfir dju ij ge ikr g fd $k = \frac{6}{\sqrt{29}}$

vr% yc oQ ikn oQ fun'kkd $\left(\frac{12}{29}, \frac{-18}{29}, \frac{24}{29}\right)$ gA

fVli.kh ;fn ey ¯cn l lery dh njh d gk vkj lery oQ vfHkyc dh fnoQ&dkl kbu l, m, n gk rc yc dk ikn (ld, md, nd) gkrk gA

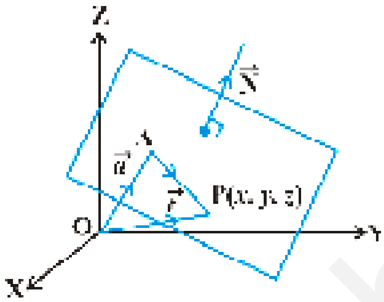
11.6.2 ,d fn, lfn'k oQ vuyc rFkk fn, ¯cn l gkdj tku oky lery dk lehdj.k *(Equation of a plane perpendicular to a given vector and passing through a given point)*

vrfj{k e] ,d fn, x, lfn'k oQ vuyc vuoQ lery gk ldr g ijr ,d fn, x, ¯cn P(x_1, y_1, z_1) l bl idkj dk oQoy ,d lery dk vfLrRo gkrk g (nf[k, vkoQfr 11-12)A



vkoQfr 11-12

eku yhft, fd lery ,d ¯cn A, ftldh fLFkfr lfn'k $\vec{a}$ g] l tkrk g vkj lfn'k $\vec{N}$ oQ vuyc gA eku yhft, fd lery ij fdlh ¯cn P dk fLFkfr lfn'k $\vec{r}$ g (vkoQfr 11-13)A



vkoQfr 11-13

rc ¯cn P lery e fLFkr gkrk g] ;fn vkj oQoy ;fn $\overrightarrow{AP}$, $\vec{N}$ ij yc g] vFkkr $\overrightarrow{AP}\cdot\vec{N}=0$. ijr $\overrightarrow{AP}=\vec{r}-\vec{a}$. blfy,

$$(\vec{r}-\vec{a})\cdot\vec{N}=0 \qquad \text{é (1)}$$

;g lery dk lfn'k lehdj.k gA

dkrh; :i **(Cartesian Form)**

eku yhft, fd fn;k ¯cn A (x_1, y_1, z_1) vkj lery ij dkb ¯cn P (x, y, z) g rFkk $\vec{N}$ oQ fnoQ&vuikr A, B rFkk C g] rc

$$\vec{a}=x_1\hat{i}+y_1\hat{j}+z_1\hat{k},\ \vec{r}=x\hat{i}+y\hat{j}+z\hat{k} \text{ vkj } \vec{N}=A\hat{i}+B\hat{j}+C\hat{k}$$

vc $(\vec{r}-\vec{a})\cdot\vec{N}=0$

blfy, $\left[(x-x_1)\hat{i}+(y-y_1)\hat{j}+(z-z_1)\hat{k}\right]\cdot(A\hat{i}+B\hat{j}+C\hat{k})=0$

vFkkr $\mathbf{A(x-x_1)+B(y-y_1)+C(z-z_1)=0}$

mnkgj.k 17 ml lery dk lfn'k vkj dkrh; lehdj.k Kkr dhft,] tk ¯cn (5, 2, ï 4) l tkrk g vkj 2, 3, ï 1 fnoQ&vuikr okyh j[kk ij yc gA

gy ge tkur g fd ¯cn (5, 2, ï 4) dk fLFkfr lfn'k $\vec{a}=5\hat{i}+2\hat{j}-4\hat{k}$ g vkj lery oQ yc dk vfHkyc lfn'k $\vec{N}=2\hat{i}+3\hat{j}-\hat{k}$ gA

blfy, lery dk lfn'k lehdj.k $(\vec{r}-\vec{a})\cdot\vec{N}=0$ l inÙk gA

;k $[\vec{r}-(5\hat{i}+2\hat{j}-4\hat{k})]\cdot(2\hat{i}+3\hat{j}-\hat{k})=0$ ... (1)

(1) dk dkrh; :i e :ikrj.k dju ij ge ikr g] fd

$$[(x-5)\hat{i}+(y-2)\hat{j}+(z+4)\hat{k}]\cdot(2\hat{i}+3\hat{j}-\hat{k})=0$$

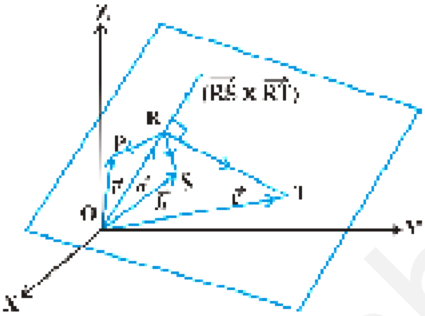
;k $2(x-5)+3(y-2)-1(z+4)=0$

vFkkr $2x+3y-z=20$

tk lery dk dkrh; lehdj.k gA

11.6.3 rhu vlj[kh; ¯cnvksa ls gksdj tkus okys lery dk lehdj.k (*Equation of a plane passing through three non-collinear points*)

eku yhft, lery ij fLFkr rhu vlj[k ¯cnvksa R, S vkSj T osQ fLFkfr lfn'k Øe'k% $\vec{a}, \vec{b}$ vkSj $\vec{c}$ gSa (vkoQfr 11.14)A



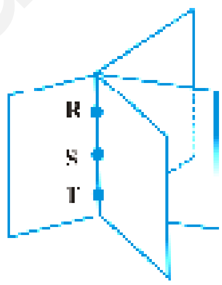
vkoQfr 11-14

lfn'k $\vec{RS}$ vkSj $\vec{RT}$ fn, lery esa gSaA blfy, lfn'k $\vec{RS} \times \vec{RT}$ ¯cnvksa R, S vkSj T dks varfoZ"V djus okys lery ij yac gksxkA eku yhft, lery esa dksbZ ¯cnq P dk fLFkfr lfn'k $\vec{r}$ gSA blfy, R ls tkus okys rFkk lfn'k $\vec{RS} \times \vec{RT}$ ij yacor~ lery dk lehdj.k $(\vec{r} - \vec{a}) \cdot (\vec{RS} \times \vec{RT}) = 0$ gSA

;k $$(\vec{r} - \vec{a}) \cdot [(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})] = 0 \quad \text{é} \quad (1)$$

;g rhu vlj[k ¯cnvksa ls xqtjus okys lery osQ lehdj.k dk lfn'k :i gSA

> **fVIi.kh** mijksDr izfØ;k esa rhu vlj[k ¯cn dgus dh vko';drk gS\ ;fn ¯cn ,d gh js[kk ij fLFkr gSa rc muls xqtjus okys db lery gksaxs (vkoQfr 11-15)A



vkoQfr 11-15

;s lery ,d iqLrd osQ i"Bksa dh Hkkfr gksaxs tgk¡ ¯cnvksa R, S vkSj T dks varfoZ"V djus okyh js[kk iqLrd osQ i"Bksa osQ c/u okys LFkku dk lnL; gSA

dkrhZ; :i (Cartesian Form)

eku yhft, ¯cnvksa R, S vkSj T osQ funsZ'kkad Øe'k% (x_1, y_1, z_1), (x_2, y_2, z_2) vkSj (x_3, y_3, z_3) gSaA eku yhft, fd lery ij fdlh ¯cn P osQ funsZ'kkad (x, y, z) o bldk fLFkfr lfn'k $\vec{r}$ gSA rc

$$\vec{RP} = (x - x_1)\hat{i} + (y - y_1)\hat{j} + (z - z_1)\hat{k}$$
$$\vec{RS} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$
$$\vec{RT} = (x_3 - x_1)\hat{i} + (y_3 - y_1)\hat{j} + (z_3 - z_1)\hat{k}$$

bu ekuksa dks lfn'k :i osQ lehdj.k (1) esa ifrLFkkiu djus ij ge ikrs gSa fd

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

tks rhu ¯cnvksa (x_1, y_1, z_1), (x_2, y_2, z_2) vkSj (x_3, y_3, z_3) ls xqtjus okys lery osQ lehdj.k dk dkrhZ; :i gSA

mnkgj.k 18 ¯cnvk R(2, 5, – 3), S(– 2, – 3, 5) vkj T(5, 3,– 3) ls tkus okys lery dk lfn'k lehdj.k Kkr dhft,A

gy eku yhft, $\vec{a}=2\hat{i}+5\hat{j}-3\hat{k}$, $\vec{b}=-2\hat{i}-3\hat{j}+5\hat{k}$, $\vec{c}=5\hat{i}+3\hat{j}-3\hat{k}$

rc $\vec{a}$, $\vec{b}$ vkj $\vec{c}$ ls tkus okys lery dk lfn'k lehdj.k fuEufyf[kr g%

$$(\vec{r}-\vec{a})\cdot(\overrightarrow{RS}\times\overrightarrow{RT})=0 \quad \text{(D;k?)}$$

;k $$(\vec{r}-\vec{a})\cdot[(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})]=0$$

vFkkZr $$[\vec{r}-(2\hat{i}+5\hat{j}-3\hat{k})]\cdot[(-4\hat{i}-8\hat{j}+8\hat{k})\times(3\hat{i}-2\hat{j})]=0$$

11.6.4 lery oQ lehdj.k dk vr% [kaM&:i *(Intercept form of the equation of a plane)*

bl vuPNsn e] ge lery oQ lehdj.k dks mloQ }kjk funsZ'kkad{kksa ij dVs vr% [kaM oQ :i esa Kkr djsaxsA eku yhft, lery dk lehdj.k

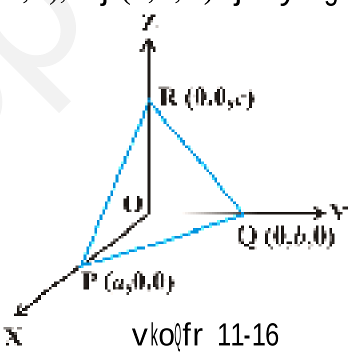
$$Ax + By + Cz + D = 0 \quad (D \neq 0) \text{ gSA} \qquad \ldots (1)$$

eku yhft, lery }kjk $x, y,$ vkj z-v{kksa ij dVs vr% [kaM Øe'k% a, b vkj c (vkoQfr 11.16) gSA Li"Vr% lery x, y vkj z-v{kksa ls Øe'k% ¯cnvksa $(a, 0, 0), (0, b, 0),$ vkj $(0, 0, c)$ ij feyrk gSA

blfy,, $Aa + D = 0$;k $A = \dfrac{-D}{a}$

$Bb + D = 0$;k $B = \dfrac{-D}{b}$

$Cc + D = 0$;k $C = \dfrac{-D}{c}$



vkoQfr 11-16

bu ekuksa dks lery oQ lehdj.k (1) esa ifrLFkkfir djus vkSj ljy djus ij ge ikrs gSa fd

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \qquad \ldots (2)$$

tks vr% [kaM :i esa lery dk vHkh"V lehdj.k gSA

mnkgj.k 19 ml lery dk lehdj.k Kkr dhft, tks x, y vkSj z-v{kksa ij Øe'k% 2] 3 vkSj 4 vr% [kaM dkVrk gSA

gy eku yhft,] lery dk lehdj.k gSA

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \qquad \ldots (1)$$

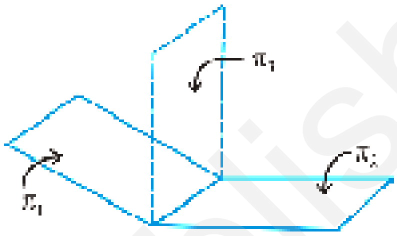
;gk¡ $a = 2, b = 3, c = 4$ Kkr gSA

a, b vkj c oQ bu ekuk dk (1) e ifrLFkkfir dju ij ge lery dk vHkh"V lehdj.k

$\frac{x}{2}+\frac{y}{3}+\frac{z}{4}=1$;k $6x + 4y + 3z = 12$ ikIr djr gA

11.6.5 nk fn, leryk oQ ifrPNnu l gkdj tku okyk lery *(Plane passing through the intersection of two given planes)*

eku yhft, π_1 vkj π_2 nk lery] ftuoQ lehdj.k Øe'k% $\vec{r}.\hat{n}_1 = d_1$ vkj $\vec{r}.\hat{n}_2 = d_2$ g buoQ ifrPNnu j[kk ij fLFkr fdlh ¯cn dk fLFkfr lfn'k bu nkuk lehdj.kk dk lr"V djxk (vko`fr 11.17)A



vko`fr 11-17

;fn bl j[kk ij fLFkr fdlh ¯cn dh fLFkfr lfn'k $\vec{t}$ g] rk

$$\vec{t}\cdot\vec{n}_1 = d_1 \text{ vkj } \vec{t}\cdot\vec{n}_2 = d_2$$

blfy, λ oQ lHkh okLrfod ekuk oQ fy, ge ikr g fd

$$\vec{t}\cdot(\vec{n}_1+\lambda\vec{n}_2) = d_1+\lambda d_2$$

D;kfd $\vec{t}$ LoPN g blfy, ;g j[kk oQ fdlh ¯cn dk lr"V djrk gA

bl idkj lehdj.k $\vec{r}\cdot(\vec{n}_1+\lambda\vec{n}_2)=d_1+\lambda d_2$ lery π_3 dk fu:fir djrk g tk ,lk g fd ;fn dkb lfn'k $\vec{r}$, π_1 vkj π_2, oQ lehdj.kk dk lr"V djrk g rk og π_3 dk vo'; lr"V djxkA

vr% leryk $\vec{r}\cdot\vec{n}_1 = d_1$ vkj $\vec{r}\cdot\vec{n}_2 = d_2$ oQ ifrPNnu j[kk l tku oky fdlh lery dk lehdj.k $\vec{r}\cdot(\vec{n}_1+\lambda\vec{n}_2) = d_1 + \lambda d_2$ gA ... (1)

dkrh; :i (Cartesian Form)

dkrh; :i oQ fy, ekuk

$$\vec{n}_1 = A_1\hat{i}+B_2\hat{j}+C_1\hat{k}$$

$$\vec{n}_2 = A_2\hat{i}+B_2\hat{j}+C_2\hat{k}$$

vkj $$\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$$

rk (1) dk ifjofrr :i g%

$$x\,(A_1 + \lambda A_2) + y\,(B_1 + \lambda B_2) + z\,(C_1 + \lambda C_2) = d_1 + \lambda d_2$$

;k $$(\mathbf{A_1}x + \mathbf{B_1}y + \mathbf{C_1}z - d_1) + \lambda(\mathbf{A_2}x + \mathbf{B_2}y + \mathbf{C_2}z - d_2) = 0 \quad \text{... (2)}$$

tk iR;d λ oQ fy, fn, leryk oQ ifrPNnu j[kk l gkdj tku oky fdlh lery dk dkrh; lehdj.k gA mnkgj.k 20 lerykas $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=6$ vkSj $\vec{r}\cdot(2\hat{i}+3\hat{j}+4\hat{k})=-5$, oQs ifrPNsnu rFkk ¯cnq $(1,1,1)$ ls tkus okys lery dk lfn'k lehdj.k Kkr dhft,A

gy ;gk¡ $\vec{n}_1=\hat{i}+\hat{j}+\hat{k}$ vkSj $\vec{n}_2=2\hat{i}+3\hat{j}+4\hat{k}$ vkSj $d_1=6$ vkSj $d_2=-5$ gSA

blfy, lw=k $\vec{r}\cdot(\vec{n}_1+\lambda\vec{n}_2)=d_1+\lambda d_2$ dk iz;ksx djus ij]

$$\vec{r}\cdot[\hat{i}+\hat{j}+\hat{k}+\lambda(2\hat{i}+3\hat{j}+4\hat{k})] = 6-5\lambda$$

;k $$\vec{r}\cdot[(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1+4\lambda)\hat{k}] = 6-5\lambda \quad \text{... (1)}$$

tgk¡ λ ,d okLrfod la[;k gSA

$\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$, j[kus ij ge ikrs gSa fd

$$(x\hat{i}+y\hat{j}+z\hat{k})\cdot[(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1+4\lambda)\hat{k}]=6-5\lambda$$

;k $$(1+2\lambda)x+(1+3\lambda)y+(1+4\lambda)z=6-5\lambda$$

;k $$(x+y+z-6)+\lambda(2x+3y+4z+5)=0 \quad \text{... (2)}$$

vc iz'ukuqlkj vHkh"V lery ¯cnq (1]1]1) ls tkrk gS] vr% ;g ¯cnq (2) dks larq"V djsxk vFkkZr

$$(1+1+1-6)+\lambda(2+3+4+5)=0$$

;k $$\lambda=\frac{3}{14}$$

λ oQs bl eku dks (1) esa ifrLFkkfir djus ij ge ikrs gSa fd

$$\vec{r}\cdot\left[\left(1+\frac{3}{7}\right)\hat{i}+\left(1+\frac{9}{14}\right)\hat{j}+\left(1+\frac{6}{7}\right)\hat{k}\right] = 6-\frac{15}{14}$$

;k $$\vec{r}\cdot\left(\frac{10}{7}\hat{i}+\frac{23}{14}\hat{j}+\frac{13}{7}\hat{k}\right) = \frac{69}{14}$$

;k $$\vec{r}\cdot(20\hat{i}+23\hat{j}+26\hat{k}) = 69$$

tks lery dk vHkh"V lfn'k lehdj.k gSA

11.7 nks js[kkvksa dk lg&ryh; gksuk (Coplanarity of two lines)

eku yhft, fd nks Kkr js[kk,¡%

$$\vec{r} = \vec{a}_1+\lambda\vec{b}_1 \quad \text{... (1)}$$

rFkk $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ g ... (2)

j[kk (1) ¯cn A, ftldh fLFkfr lfn'k $\vec{a}_1$ g] l gkdj tkrh g rFkk $\vec{b}_1$ o lekrj gA j[kk (2) ¯cn B ftldh fLFkfr lfn'k $\vec{a}_2$ g] l gkdj tkrh g rFkk $\vec{b}_2$ o lekrj gA rc

$$\overrightarrow{AB} = \vec{a}_2 - \vec{a}_1$$

Kkr j[kk, lg&ryh; g] ;fn vkj ooy ;fn $\overrightarrow{AB}, \vec{b}_1$ vkj $\vec{b}_2$ lg&ryh; gA vFkkr

$$\overrightarrow{AB}.(\vec{b}_1 \times \vec{b}_2) = 0 \text{ ;k } (\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2) = 0$$

dkrh; :i **(Cartesian Form)**

eku yhft, fd ¯cnvk A vkj B o fun'kkd Øe'k% (x_1, y_1, z_1) vkj (x_2, y_2, z_2) gA eku yhft, fd $\vec{b}_1$ vkj $\vec{b}_2$ o fnoQ&vuikr Øe'k% a_1, b_1, c_1 rFkk a_2, b_2, c_2 gA rc

$$\overrightarrow{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$\vec{b}_1 = a_1\hat{i} + b_1\hat{j} + c_1\hat{k} \text{ vkj } \vec{b}_2 = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$$

Kkr j[kk, lg&ryh; g] ;fn vkj ooy ;fn $\overrightarrow{AB}\cdot(\vec{b}_1 \times \vec{b}_2) = 0$ ftl fuEufyf[kr dkrh; :i e O;Dr dj ldr gA

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0 \quad \text{... (4)}$$

mnkgj.k **21** n'kkb, fd j[kk,

$$\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5} \text{ rFkk } \frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5} \text{ lg\&ryh; gA}$$

gy ;gk ge Kkr g fd $x_1 = -3, y_1 = 1, z_1 = 5, a_1 = -3, b_1 = 1, c_1 = 5$

$$x_2 = -1, y_2 = 2, z_2 = 5, a_2 = -1, b_2 = 2, c_2 = 5$$

vc fuEufyf[kr lkjf.kd yu ij ge ikr g fd

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 2 & 1 & 0 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix} = 0$$

blfy, j[kk, le&ryh; gA

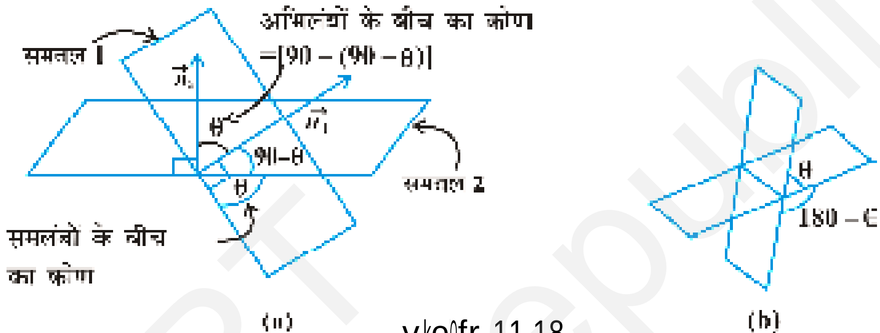
11.8 nk lery k oQ chp dk dk.k (Angle between two planes)

ifjHkk"kk 2 nk lery k oQ chp dk dk.k muoQ vfHkyck oQ eè;LFk dk.k }kjk ifjHkkf"kr g (vko`Qfr 11.18 (a))A è;ku nhft, fd ;fn nk lery k oQ chp dk dk.k θ g rk 180 ï θ (vko`Qfr 11.18 (b)) Hkh muoQ chp dk dk.k gA ge U;u dk.k dk gh lerykoQ chp dk dk.k yxA

eku yhft, fd leryk] $\vec{r}\cdot\vec{n}_1 = d_1$ vkj $\vec{r}\cdot\vec{n}_2 = d_2$ oQ chp dk dk.k θ gA rc fdlh lko ¯cnq l leryk ij [khp x, vfHkyck oQ chp dk dk.k θ gA

rc

$$\cos\theta = \left|\frac{\vec{n}_1\cdot\vec{n}_2}{|\vec{n}_1|\,|\vec{n}_2|}\right|$$



vko`Qfr 11-18

> **fVIi.kh** nksuk lery ijLij yacor g ;fn $\vec{n}_1 . \vec{n}_2 = 0$ vkj lekarj g ;fn $\vec{n}_1$ vkj $\vec{n}_2$ lekarj gA

dkrh; :i (Cartesian Form)

eku yhft, leryk

$$A_1 x + B_1 y + C_1 z + D_1 = 0 \text{ vkj } A_2 x + B_2 y + C_2 z + D_2 = 0$$

oQ chp dk dk.k θ gA

rk leryk oQ vfHkyc oQ fnoQ&vuikr Øe'k% A_1, B_1, C_1 vkj A_2, B_2, C_2 gA blfy,

$$\cos\theta = \left|\frac{A_1 A_2 + B_1 B_2 + C_1 C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2}\ \sqrt{A_2^2 + B_2^2 + C_2^2}}\right|$$

> **fVIi.kh**
>
> 1. ;fn nksuk lery ijLij yc g rc θ = 90° vkj blfy, cos θ = 0. vr% $\cos\theta = A_1A_2 + B_1B_2 + C_1C_2 = 0$

2. ;fn nkuk lery lekrj g rk $\dfrac{A_1}{A_2} = \dfrac{B_1}{B_2} = \dfrac{C_1}{C_2}$

mnkgj.k 22 nk leryk $2x + y - 2z = 5$ vkj $3x - 6y - 2z = 7$ o chp dk dk.k lfn'k fof/ }kjk Kkr dhft,A

gy nk leryk o chp dk dk.k ogh g tk muo vfHkyck o chp dk dk.k gA leryk o fn, x, lehdj.kk l leryk o lfn'k vfHkyc

$$\vec{N}_1 = 2\hat{i} + \hat{j} - 2\hat{k} \text{ vkj } \vec{N}_2 = 3\hat{i} - 6\hat{j} - 2\hat{k} \text{ gA}$$

blfy,
$$\cos\theta = \left|\frac{\vec{N}_1 \cdot \vec{N}_2}{|\vec{N}_1|\,|\vec{N}_2|}\right| = \left|\frac{(2\hat{i} + \hat{j} - 2\hat{k})\cdot(3\hat{i} - 6\hat{j} - 2\hat{k})}{\sqrt{4+1+4}\ \sqrt{9+36+4}}\right| = \left(\frac{4}{21}\right)$$

vr%
$$\theta = \cos^{-1}\left(\frac{4}{21}\right)$$

mnkgj.k 23 nk leryk $3x - 6y + 2z = 7$ vkj $2x + 2y - 2z = 5$ o chp dk dk.k Kkr dhft,A

gy leryk dh Kkr lehdj.kk dh ryuk lehdj.kk

$$A_1 x + B_1 y + C_1 z + D_1 = 0 \text{ vkj } A_2 x + B_2 y + C_2 z + D_2 = 0$$

l djus ij ge ikr g fd%
$$A_1 = 3,\ B_1 = -6,\ C_1 = 2$$
$$A_2 = 2,\ B_2 = 2,\ C_2 = -2$$

iu%
$$\cos\theta = \left|\frac{3\times 2 + (-6)\,(2) + (2)\,(-2)}{\sqrt{\left(3^2 + (-6)^2 + (-2)^2\right)}\ \sqrt{\left(2^2 + 2^2 + (-2)^2\right)}}\right|$$

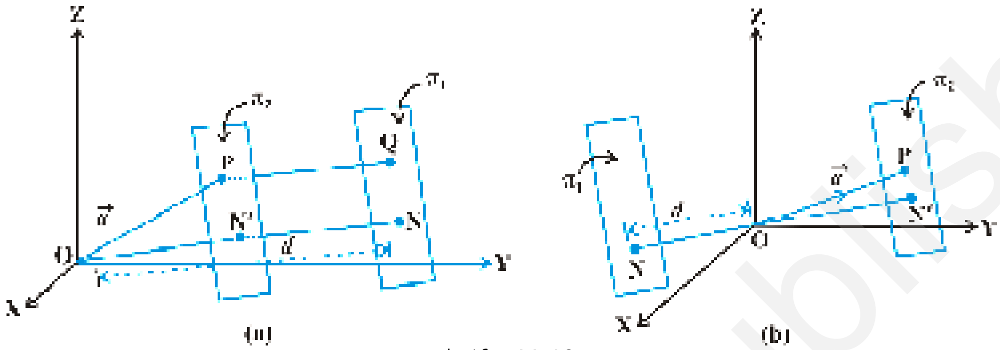
$$= \left|\frac{-10}{7\times 2\sqrt{3}}\right| = \frac{5}{7\sqrt{3}} = \frac{5\sqrt{3}}{21}$$

blfy,
$$\theta = \cos^{-1}\left(\frac{5\sqrt{3}}{21}\right)$$

11.9 lery l fn, x, ¯cn dh njh (Distance of a point from a plane)

lfn'k :i (Vector Form)

,d ¯cn P ftldk fLFkfr lfn'k $\vec{a}$ vkj ,d lery π_1 ftldk lehdj.k $\vec{r}\cdot\hat{n} = d$ (vkoQfr 11.19) ij fopkj dhft,A



vkoQfr 11-19

iuÚ ¯cn P l lery π_1 oQ lekrj lery π_2 ij fopkj dhft,A lery π_2 oQ vfHkyc bdkb lfn'k $\hat{n}$ gA vr% bldk lehdj.k $(\vec{r} - \vec{a})\cdot\hat{n} = 0$ gA

vFkkr $\vec{r}\cdot\hat{n} = \vec{a}\cdot\hat{n}$

vr%] ey ¯cn l bl lery dh njh ON′ = $|\vec{a}\cdot\hat{n}|$ gA blfy, P l lery π_1 l njh (vkoQfr 11.21 (a))

$$PQ = ON - ON' = |d - \vec{a}\cdot\hat{n}|$$

g] tk ,d ¯cn l Kkr lery ij yc dh ycbZ gA vkoQfr 11.19 (b) oQ fy, ge blh idkj dk ifj.kke LFkkfir dj ldr gA

fVli.kh

1. ;fn lery π_2 dk lehdj.k $\vec{r}\cdot\vec{N} = d$, oQ :i dk g] tgk $\vec{N}$ lery ij vfHkyc g rk ykfcd njh $\dfrac{|\vec{a}\cdot\vec{N} - d|}{|\vec{N}|}$ gA

2. ey ¯cn O l lery $\vec{r}\cdot\vec{N} = d$ dh njh $\dfrac{|d|}{|\vec{N}|}$ g (D;kfd $\vec{a} = 0$)A

dkrh; :i **(Cartesian Form)**

eku yhft, fd $P(x_1, y_1, z_1)$,d fn;k ¯cnq gS ftldk fLFkfr lfn'k $\vec{a}$ gS rFkk fn, lery dk dkrh; lehdj.k

$$Ax + By + Cz = D \text{ gS}$$

rc $$\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$

$$\vec{N} = A\hat{i} + B\hat{j} + C\hat{k}$$

vr% (1) o) }kjk P ls lery ij yac dh yackbZ

$$\left| \frac{(x_1\hat{i} + y_1\hat{j} + z_1\hat{k}) \cdot (A\hat{i} + B\hat{j} + C\hat{k}) - D}{\sqrt{A^2 + B^2 + C^2}} \right|$$

$$= \left| \frac{A x_1 + B y_1 + C z_1 - D}{\sqrt{A^2 + B^2 + C^2}} \right|$$

mnkgj.k **24** ¯cnq (2, 5, – 3) dh lery $\vec{r} \cdot (6\hat{i} - 3\hat{j} + 2\hat{k}) = 4$ ls njh Kkr dhft,A

gy ;gk¡ $\vec{a} = 2\hat{i} + 5\hat{j} - 3\hat{k}$, $\vec{N} = 6\hat{i} - 3\hat{j} + 2\hat{k}$ vkSj $d = 4$.

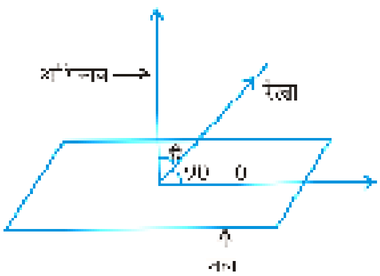
blfy, ¯cnq (2, 5, – 3) dh fn, lery ls njh gS

$$\frac{|(2\hat{i} + 5\hat{j} - 3\hat{k}) \cdot (6\hat{i} - 3\hat{j} + 2\hat{k}) - 4|}{|6\hat{i} - 3\hat{j} + 2\hat{k}|}$$

$$= \frac{|12 - 15 - 6 - 4|}{\sqrt{36 + 9 + 4}} = \frac{13}{7}$$

11.10 ,d js[kk vkSj ,d lery osQ chp dk dks.k (Angle between a line and a plane)

ifjHkk"kk **2** ,d js[kk vkSj ,d lery osQ chp dk dks.k] js[kk vkSj lery osQ vfHkyac osQ chp osQ dks.k dk dks.k (complementary angle) ijd gksrk gS (vko`Qfr 11.20)A



vko`Qfr 11-20

lfn'k :i **(Vector Form)**

eku yhft, fd js[kk dk lehdj.k $\vec{r} = \vec{a} + \lambda\vec{b}$ gS rFkk lery dk lehdj.k $\vec{r} \cdot \vec{n} = d$ gSA rc js[kk vkSj lery osQ vfHkyc o‚ chp dk dk.k θ, fuEufyf[kr l=k }kjk 0;Dr fd;k tk ldrk gA

$$\cos\theta = \left|\frac{\vec{b}\cdot\vec{n}}{|\vec{b}|\cdot|\vec{n}|}\right|$$

vkj bl idkj j[kk vkj lery o‚ chp dk dk.k ϕ, $90° - \theta$, }kjk inÙk g vFkkr

$$\sin(90° - \theta) = \cos\theta$$

vFkkr] $$\sin\phi = \left|\frac{\vec{b}\cdot\vec{n}}{|\vec{b}|\;|\vec{n}|}\right| \quad ;k \quad \phi = \sin^{-1}\left|\frac{\vec{b}\cdot\vec{n}}{|\vec{b}||\vec{n}|}\right|$$

mnkgj.k 25 j[kk $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ vkj lery $10x + 2y - 11z = 3$ o‚ chp dk dk.k Kkr dhft,A

gy eku yhft, fd j[kk vkj lery o‚ vfHkyc o‚ chp dk dk.k θ gA fn, x, j[kk rFkk lery o‚ lehdj.kk dk lfn'k :i e 0;Dr dju ij ge

$$\vec{r} = (-\hat{i} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

vkj $\vec{r}\cdot(10\hat{i} + 2\hat{j} - 11\hat{k}) = 3$ iklr djr gA

;gk $\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ vkj $\vec{n} = 10\hat{i} + 2\hat{j} - 11\hat{k}$

vr% $$\sin\phi = \left|\frac{(2\hat{i}+3\hat{j}+6\hat{k})\cdot(10\hat{i}+2\hat{j}-11\hat{k})}{\sqrt{2^2+3^2+6^2}\;\sqrt{10^2+2^2+11^2}}\right|$$

$$= \left|\frac{-40}{7\times 15}\right| = \left|\frac{-8}{21}\right| = \frac{8}{21} \quad ;k \quad \phi = \sin^{-1}\left(\frac{8}{21}\right)$$

i'ukoyh 11-3

1. fuEufyf[kr i'uk e l iR;d e lery o‚ vfHkyc dh fno‚&dkT;kbu vkj ey ¯cn l njh Kkr dhft,%
   (a) $z = 2$ (b) $x + y + z = 1$
   (c) $2x + 3y - z = 5$ (d) $5y + 8 = 0$
2. ml lery dk lfn'k lehdj.k Kkr dhft,] tk ey ¯cn l 7 ek=kd njh ij g] vkj lfn'k $3\hat{i} + 5\hat{j} - 6\hat{k}$ ij vfHkyc gA

3. fuEufyf[kr lery dk dkrh; lehdj.k Kkr dhft,%

(a) $\vec{r}\cdot(\hat{i}+\hat{j}-\hat{k})=2$ (b) $\vec{r}\cdot(2\hat{i}+3\hat{j}-4\hat{k})=1$

(c) $\vec{r}\cdot[(s-2t)\hat{i}+(3-t)\hat{j}+(2s+t)\hat{k}]=15$

4. fuEufyf[kr fLFkfr;k e] ey ¯cn l [khp x, yc oQ ikn oQ fun'kkad Kkr dhft,A

(a) $2x+3y+4z-12=0$ (b) $3y+4z-6=0$

(c) $x+y+z=1$ (d) $5y+8=0$

5. fuEufyf[kr ifrc/k oQ vrxr lery dk lfn'k ,o dkrh; lehdj.k Kkr dhft, tk%

(a) ¯cn $(1, 0, -2)$ l tkrk gk vkj $\hat{i}+\hat{j}-\hat{k}$ lery ij vfHkyc gA

(b) ¯cn $(1,4, 6)$ l tkrk gk vkj $\hat{i}-2\hat{j}+\hat{k}$ lery ij vfHkyc lfn'k gA

6. mu lery dk lehdj.k Kkr dhft, tk fuEufyf[kr rhu ¯cnvk l xtjrk gA

(a) $(1, 1, -1), (6, 4, -5), (-4, -2, 3)$

(b) $(1, 1, 0), (1, 2, 1), (-2, 2, -1)$

7. lery $2x+y-z=5$ }kjk dkV x, vr% [kMk dk Kkr dhft,A

8. ml lery dk lehdj.k Kkr dhft, ftldk y&v{k ij vr%[kM 3 vkj tk ry ZOX oQ lekrj gA

9. ml lery dk lehdj.k Kkr dhft, tk leryk $3x-y+2z-4=0$ vkj $x+y+z-2=0$ oQ ifrPNnu rFkk ¯cn $(2, 2, 1)$ l gkdj tkrk gA

10. ml lery dk lfn'k lehdj.k Kkr dhft, tk leryk $\vec{r}.(2\hat{i}+2\hat{j}-3\hat{k})=7$, $\vec{r}.(2\hat{i}+5\hat{j}+3\hat{k})=9$ oQ ifrPNnu j[kk vkj $(2, 1, 3)$ l gkdj tkrk gA

11. ryk $x+y+z=1$ vkj $2x+3y+4z=5$ oQ ifrPNnu j[kk l gkdj tku oky rFkk ry $x-y+z=0$ ij ycor ry dk lehdj.k Kkr dhft,A

12. leryk] ftuoQ lfn'k lehdj.k $\vec{r}\cdot(2\hat{i}+2\hat{j}-3\hat{k})=5$ vkj $\vec{r}\cdot(3\hat{i}-3\hat{j}+5\hat{k})=3$ g] oQ chp dk dk.k Kkr dhft,A

13. fuEufyf[kr i'uk e Kkr dhft, fd D;k fn, x, leryk oQ ;Xe lekrj g vFkok ycor g] vkj ml fLFkfr e] tc ; u rk lekrj g vkj u gh ycor rk muoQ chp dk dk.k Kkr dhft,A

(a) $7x+5y+6z+30=0$ vkj $3x-y-10z+4=0$

(b) $2x+y+3z-2=0$ vkj $x-2y+5=0$

(c) $2x-2y+4z+5=0$ vkj $3x-3y+6z-1=0$

(d) $2x-y+3z-1=0$ vkj $2x-y+3z+3=0$

(e) $4x+8y+z-8=0$ vkj $y+z-4=0$

14. fuEufyf[kr i'uk e iR;d fn, x, ¯cn l fn, x, lxr lerykdh njh Kkr dhft,A

	¯cn	lery
(a)	$(0, 0, 0)$	$3x - 4y + 12z = 3$
(b)	$(3, -2, 1)$	$2x - y + 2z + 3 = 0$
(c)	$(2, 3, -5)$	$x + 2y - 2z = 9$
(d)	$(-6, 0, 0)$	$2x - 3y + 6z - 2 = 0$

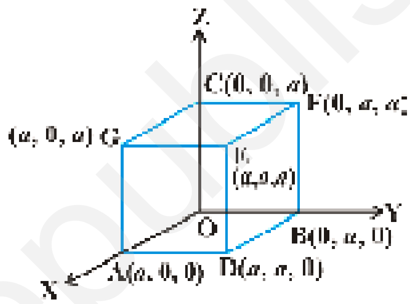
fofo/ mnkgj.k

mnkgj.k 26 ,d j[kk] ,d ?ku oQ fod.kk oQ lkFk $\alpha, \beta, \gamma, \delta$, dk.k cukrh g rk fl¼ dhft, fd

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2 \delta = \frac{4}{3}$$

gy ,d ?ku] ,d ledkf.kd "kVi`yd; gkrk g ftl dh ycb] pkMkb vkj Åpkb leku gkr gA

eku yhft, fd OADBEFCG ,d ?ku ftl dh iR;d Hktk a ycb dh g (vko`fr 11.21)A



vko`fr 11-21

OE, AF, BG vkj CD pkj fod.k gA

nk ¯cnvk O rFkk E dk feyku okyh j[kk OE vFkkr fod.k OE oQ fno`Q&dkl kbu

$$\frac{a-0}{\sqrt{a^2+a^2+a^2}}, \frac{a-0}{\sqrt{a^2+a^2+a^2}}, \frac{a-0}{\sqrt{a^2+a^2+a^2}}$$

vFkkr $\quad \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

gA blh idkj AF, BG vkj CD dh fno`Q&dkl kbu Øe'k%

$$-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}; \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \text{ vkj } \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \text{gA}$$

eku yhft, nh xb j[kk tk OE, AF, BG, vkj CD, oQ lkFk Øe'k% α, β, γ, vkj δ dk.k cukrh g] dh fno`Q&dkl kbu l, m, n gA

rc $\quad \cos\alpha = \frac{1}{\sqrt{3}}(l + m + n); \cos\beta = \frac{1}{\sqrt{3}}(-l + m + n)$

$$\cos\gamma = \frac{1}{\sqrt{3}}(l - m + n); \cos\delta = \frac{1}{\sqrt{3}}(l + m - n)$$

ox djoQ tkMu ij ge ikr g fd

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta$$

$$= \frac{1}{3}\ [\,(l + m + n\,)^2 + (-\,l + m + n)^2\,]\ + (l - m + n)^2 + (l + m - n)^2]$$

$$= \frac{1}{3}\ [\ 4\ (l^2 + m^2 + n^2\,)\ ]\ = \frac{4}{3}$$ (D;kfd $l^2 + m^2 + n^2 = 1$)

mnkgj.k 27 mI ry dk lehdj.k Kkr dhft, ftlesa ¯cn $(1, -1, 2)$ vrfo"V g vkj tks leryksa $2x + 3y - 2z = 5$ vkj $x + 2y - 3z = 8$ e l iR;d ij yc gA

gy fn, x, ¯cn dks vrfo"V dju oky leryksa dk lehdj.k

$$A\,(x - 1) + B(y + 1) + C\,(z - 2) = 0 \text{ gA} \qquad \ldots (1)$$

leryksa $2x + 3y - 2z = 5$ vkj $x + 2y - 3z = 8$, oQ lkFk (1) }kjk inÙk lery ij yc gku oQ ifrc/ dks i;kx dju ij ge ikr g fd

$$2A + 3B - 2C = 0 \text{ vkj } A + 2B - 3C = 0$$

bu lehdj.kksa dks gy dju ij ge ikr g fd $A = -5C$ vkj $B = 4C$

vr% vHkh"V lehdj.k g%

$$-5C\,(x - 1) + 4\,C\,(y + 1) + C(z - 2) = 0$$

vFkkr $5x - 4y - z = 7$

mnkgj.k 28 ¯cn $P(6, 5, 9)$ l ¯cnvksa $A\,(3, -1, 2)$, $B\,(5, 2, 4)$ vkj $C(-1, -1, 6)$ }kjk fu/kfjr lery dh njh Kkr dhft,A

gy eku yhft, fd lery e rhu ¯cn A, B, rFkk C gA ¯cn P l lery ij yc dk ikn D gA ge vHkh"V njh PD Kkr djuh g tgk PD, $\overrightarrow{AP}$ dk $\overrightarrow{AB} \times \overrightarrow{AC}$ ij i{ki gA

vr% PD = $\overrightarrow{AB} \times \overrightarrow{AC}$ oQ vufn'k bdkb lfn'k rFkk $\overrightarrow{AP}$ dk vfn'k x.kuiQy gA

iu% $\overrightarrow{AP} = 3\,\hat{i} + 6\,\hat{j} + 7\,\hat{k}$

vkj
$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 2 \\ -4 & 0 & 4 \end{vmatrix} = 12\hat{i} - 16\hat{j} + 12\hat{k}$$

$\overrightarrow{AB} \times \overrightarrow{AC}$ oQ vufn'k bdkb lfn'k $= \dfrac{3\hat{i} - 4\hat{j} + 3\hat{k}}{\sqrt{34}}$

vr% $$\overrightarrow{PD} = (3\hat{i}+6\hat{j}+7\hat{k})\cdot\frac{3\hat{i}-4\hat{j}+3\hat{k}}{\sqrt{34}}$$

$$= \frac{3\sqrt{34}}{17}$$

fodYir% ¯cn A, B vkj C l x P

dh lery l njh Kkr dhft,A

mnkgj.k **29** n'kkb, fd j[kk,

$$\frac{x-a+d}{\alpha-\delta} = \frac{y-a}{\alpha} = \frac{z-a-d}{\alpha+\delta}$$

vkj $$\frac{x-b+c}{\beta-\gamma} = \frac{y-b}{\beta} = \frac{z-b-c}{\beta+\gamma}$$ lg&ryh; gA

gy ;gk Kkr g fd

$x_1 = a \text{ ï } d$ vkj $x_2 = b \text{ ï } c$

$y_1 = a$ $y_2 = b$

$z_1 = a + d$ $z_2 = b + c$

vkj $a_1 = \alpha \text{ ï } \delta$ $a_2 = \beta \text{ ï } \gamma$

$b_1 = \alpha$ $b_2 = \beta$

$c_1 = \alpha + \delta$ $c_2 = \beta + \gamma$

vc lkjf.kd

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} b-c-a+d & b-a & b+c-a-d \\ \alpha-\delta & \alpha & \alpha+\delta \\ \beta-\gamma & \beta & \beta+\gamma \end{vmatrix}$$

ij fopkj dhft,A

rhlj LrHk dk igy LrHk e tkMu ij ge ikr gA

$$2\begin{vmatrix} b-a & b-a & b+c-a-d \\ \alpha & \alpha & \alpha+\delta \\ \beta & \beta & \beta+\gamma \end{vmatrix} = 0$$

D;kfd iFke vkj f}rh; LrHk leku gA vr% nkuk j[kk, lg&ryh; gA mnkgj.k 30 mI ¯cn oQ fun'kZd Kkr dhft, tgk¡ ¯cnvksa A(3, 4, 1) vkSj B(5, 1, 6) dks feykus okyh js[kk XY-ry dks dkVrh gSA

gy ¯cnvksa A vkSj B ls tkus okyh js[kk dk lfn'k lehdj.k%

$$\vec{r} = 3\hat{i} + 4\hat{j} + \hat{k} + \lambda\,[(5-3)\hat{i} + (1-4)\hat{j} + (6-1)\hat{k}]$$

vFkkZr $\quad \vec{r} = 3\hat{i} + 4\hat{j} + \hat{k} + \lambda\,(2\hat{i} - 3\hat{j} + 5\hat{k})$ ... (1)

eku yhft, P og ¯cnq gS tgk¡ js[kk AB, XY-ry dks izfrPNsn djrh gSA rc ¯cnq P dk fLFkfr lfn'k $x\hat{i} + y\hat{j}$ osQ :i esa gSA

;g ¯cnq vo'; gh lehdj.k (1) dks larq"V djrk gSA (D;ksa?)

vFkkZr $\quad x\hat{i} + y\hat{j} = (3 + 2\lambda)\hat{i} + (4 - 3\lambda)\hat{j} + (1 + 5\lambda)\hat{k}$

$\hat{i}, \hat{j}$ vkSj $\hat{k}$, osQ x.kkadksa dh rqyuk djus ij ge ikrs gSa

$$x = 3 + 2\lambda$$
$$y = 4 - 3\lambda$$
$$0 = 1 + 5\lambda$$

mijksDr lehdj.kksa dks gy djus ij ge ikrs gSa fd

$$x = \frac{13}{5} \text{ vkSj } y = \frac{23}{5}$$

vr% vHkh"V ¯cnq osQ fun'kZd $\left(\frac{13}{5}, \frac{23}{5}, 0\right)$ gSA

vè;k; 11 ij fofo/ iz'ukoyh

1. fn[kkb,, fd ewy ¯cnq ls (2, 1, 1) feykus okyh js[kk] ¯cnqvksa (3, 5, &1) vkSj (4, 3, &1) ls fu/kZfjr js[kk ij yac gSA
2. ;fn nks ijLij yac js[kkvksa dh fnd~&dksT;k,¡ l_1, m_1, n_1 vkSj l_2, m_2, n_2 gksa rks fn[kkb, fd bu nksuksa ij yac js[kk dh fnd~&dksT;k,¡ $m_1 n_2 - m_2 n_1,\ n_1 l_2 - n_2 l_1,\ l_1 m_2 - l_2 m_1$ gSA
3. mu js[kkvksa osQ eè; dks.k Kkr dhft,] ftuosQ fnd~&vuqikr a, b, c vkSj $b - c, c - a, a - b$ gSA
4. x-v{k osQ lekarj rFkk ewy&¯cnq ls tkus okyh js[kk dk lehdj.k Kkr dhft,A
5. ;fn ¯cnqvksa A, B, C, vkSj D osQ fun'kZd Øe'k% (1, 2, 3), (4, 5, 7), (– 4, 3, – 6) vkSj (2, 9, 2) gSa rks AB vkSj CD js[kkvksa osQ chp dk dks.k Kkr dhft,A

6. ;fn j[kk,¡ $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ vkSj $\frac{x-1}{3k}=\frac{y-1}{1}=\frac{z-6}{-5}$ ijLij yEc gksa rks k dk eku Kkr dhft,A

7. fcanq $(1, 2, 3)$ ls tkus okyh rFkk ry $\vec{r}\cdot(\hat{i}+2\hat{j}-5\hat{k})+9=0$ ij yEcor j[kk dk lfn'k lehdj.k Kkr dhft,A

8. fcanq (a, b, c) ls tkus okys rFkk ry $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=2$ osQ lekarj ry dk lehdj.k Kkr dhft,A

9. j[kkvksa $\vec{r}=6\hat{i}+2\hat{j}+2\hat{k}+\lambda(\hat{i}-2\hat{j}+2\hat{k})$ vkSj

$\vec{r}=-4\hat{i}-\hat{k}+\mu(3\hat{i}-2\hat{j}-2\hat{k})$ osQ chp dh U;wure nwjh Kkr dhft,A

10. ml fcanq osQ funsZ'kkad Kkr dhft, tgk¡ fcanqvksa $(5, 1, 6)$ vkSj $(3, 4, 1)$ dks feykus okyh j[kk YZ-ry dks dkVrh gSA

11. ml fcanq osQ funsZ'kkad Kkr dhft, tgk¡ fcanqvksa $(5, 1, 6)$ vkSj $(3, 4, 1)$ dks feykus okyh j[kk ZX-ry dks dkVrh gSA

12. ml fcanq osQ funsZ'kkad Kkr dhft, tgk¡ fcanqvksa $(3, -4, -5)$ vkSj $(2, -3, 1)$ ls xqtjus okyh j[kk] lery $2x+y+z=7$ osQ ikj tkrh gSA

13. fcanq $(-1, 3, 2)$ ls tkus okys rFkk lerykas $x+2y+3z=5$ vkSj $3x+3y+z=0$ esa ls izR;sd ij yac lery dk lehdj.k Kkr dhft,A

14. ;fn fcanq $(1, 1, p)$ vkSj $(-3, 0, 1)$ lery $\vec{r}\cdot(3\hat{i}+4\hat{j}-12\hat{k})+13=0$ ls leku nwjh ij fLFkr gksa] rks p dk eku Kkr dhft,A

15. lerykas $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=1$ vkSj $\vec{r}\cdot(2\hat{i}+3\hat{j}-\hat{k})+4=0$ osQ ifrPNsnu j[kk ls tkus okys rFkk x-v{k osQ lekarj ry dk lehdj.k Kkr dhft,A

16. ;fn O ewy fcanq rFkk fcanq P osQ funsZ'kkad $(1, 2, -3)$, gS rks fcanq P ls tkus okys rFkk OP osQ yEcor ry dk lehdj.k Kkr dhft,A

17. lerykas $\vec{r}\cdot(\hat{i}+2\hat{j}+3\hat{k})-4=0$ vkSj $\vec{r}\cdot(2\hat{i}+\hat{j}-\hat{k})+5=0$ osQ ifrPNsnu j[kk dks varfoZ"V djus okys rFkk ry $\vec{r}\cdot(5\hat{i}+3\hat{j}-6\hat{k})+8=0$ osQ yEcor ry dk lehdj.k Kkr dhft,A

18. fcanq $(-1, -5, -10)$ ls j[kk $\vec{r}=2\hat{i}-\hat{j}+2\hat{k}+\lambda(3\hat{i}+4\hat{j}+2\hat{k})$ vkSj lery $\vec{r}\cdot(\hat{i}-\hat{j}+\hat{k})=5$ osQ ifrPNsnu fcanq osQ e/; dh nwjh Kkr dhft,A

19. ¯cn (1, 2, 3) I tku okyh rFkk lery $\vec{r}\cdot(\hat{i}-\hat{j}+2\hat{k})=5$ vkj $\vec{r}\cdot(3\hat{i}+\hat{j}+\hat{k})=6$ oQ lekrj j[kk dk lfn'k lehdj.k Kkr dhft,A

20. ¯cn (1, 2, ï 4) I tku okyh vkj nkuk j[kvk $\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7}$ vkj $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$ ij yc j[kk dk lfn'k lehdj.k Kkr dhft,A

21. ;fn ,d lery oQ vr%[kM a, b, c g vkj bldh ey ¯cn I njh p bdkb g rk fl¼ dhft, fd $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{p^2}$

i'uk 22 vkj 23 e lgh mÙkj dk puko dhft,A

22. nk leryk $2x+3y+4z=4$ vkj $4x+6y+8z=12$ oQ chp dh njh g%

(A) 2 bdkb (B) 4 bdkb (C) 8 bdkb (D) $\frac{2}{\sqrt{29}}$ bdkb

23. lery $2x - y + 4z = 5$ vkj $5x - 2.5y + 10z = 6$ g%

(A) ijLij yc (B) lekrj

(C) y-v{k ij ifrPNnu djr gA (D) ¯cn $\left(0, 0, \frac{5}{4}\right)$ I xtjr gA

lkjk'k

- ,d j[kk dh fnoQ&dkIkbu j[kk }kjk fun'kk{kk dh /u fn'kk oQ IkFk cuk, dk.kk dh dkIkbu gkrh gA
- ;fn ,d j[kk dh fnoQ&dkIkbu l, m, n g rk $l^2+m^2+n^2=1$
- nk ¯cnvk $P(x_1, y_1, z_1)$ vkj $Q(x_2, y_2, z_2)$ dk feyku okyh j[kk dh fnoQ&dkIkbu

$$\frac{x_2-x_1}{PQ}, \frac{y_2-y_1}{PQ}, \frac{z_2-z_1}{PQ}\ \text{g}$$

tgk $PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$
- ,d j[kk dk fnoQ&vuikr o I[;k, g tk j[kk dh fnoQ&dkIkbu oQ lekuikrh gkrh gA

- ;fn ,d j[kk dh fnoQ&dkI kbu l, m, n vkj fnoQ&vuikr a, b, c g rk

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}}\ ;\ m = \frac{b}{\sqrt{a^2 + b^2 + c^2}}\ ;\ n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

- fo"kery; j[kk, vrfj{k dh o j[kk, tk u rk lekrj g vkj u gh ifrPNnh gA ;g j[kk, fofHkUu ryk e gkrh gA
- fo"kery; j[kkvk oQ chp dk dk.k og dk.k g tk ,d fdlh ¯cn (ojh;rk ey ¯cn dh) l fo"kery; j[kkvk e l iR;d oQ lekrj [khph xb nk ifrPNnh j[kkvk oQ chp e gA
- ;fn l_1, m_1, n_1 vkj l_2, m_2, n_2 fnoQ&dkI kbu okyh nk j[kkvk oQ chp U;udk.k θ g rc

$$\cos\theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|$$

- ;fn a_1, b_1, c_1 vkj a_2, b_2, c_2 fnoQ&vuikrk okyh nk j[kkvk oQ chp dk U;u dk.k θ g rc

$$\cos\theta = \left|\frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\ \sqrt{a_2^2 + b_2^2 + c_2^2}}\right|$$

- ,d Kkr ¯cn ftldh fLFkfr lfn'k $\vec{a}$ g l x $\vec{b}$ oQ lekrj j[kk dk lfn'k lehdj.k $\vec{r} = \vec{a} + \lambda \vec{b}$ gA
- ¯cn (x_1, y_1, z_1) l tku okyh j[kk ftldh fnoQ&dkI kbu l, m, n g] dk lehdj.k

$$\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$$ gA

- nk ¯cnvk ftuoQ fLFkfr lfn'k $\vec{a}$ vkj $\vec{b}$ g l tku okyh j[kk oQ lehdj.k dk lfn'k lehdj.k $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$ gA
- nk ¯cnvk (x_1, y_1, z_1) vkj (x_2, y_2, z_2) l tku okyh j[kk dk dkrh; lehdj.k

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$ gA

- ;fn nk j[kkvk $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ vkj $\vec{r} = \vec{a}_2 + \lambda \vec{b}_2$, oQ chp dk U;udk.k θ g rk

$$\cos\theta = \left|\frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1|\,|\vec{b}_2|}\right|$$

- ;fn nk j[kkvk $\frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{n_1}$ vkj

  $\frac{x-x_2}{l_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{n_2}$ oQ chp dk dk.k θ g rc

  $\cos\theta = |l_1l_2+m_1m_2+n_1n_2|$.

- nk fo"keryh; j[kkvk oQ chp dh U;ure njh og j[kk[kM g tk nkuk j[kkvk ij yc gA

- nk j[kkvk $\vec{r}=\vec{a}_1+\lambda\vec{b}_1$ vkj $\vec{r}=\vec{a}_2+\mu\vec{b}_2$ oQ chp U;ure njh

$$\left|\frac{(\vec{b}_1\times\vec{b}_2)\cdot(\vec{a}_2-\vec{a}_1)}{|\vec{b}_1\times\vec{b}_2|}\right| \text{ gA}$$

- nk j[kkvk $\frac{x-x_1}{a_1}=\frac{y-y_1}{b_1}=\frac{z-z_1}{c_1}$ vkj $\frac{x-x_2}{a_2}=\frac{y-y_2}{b_2}=\frac{z-z_2}{c_2}$ oQ chp U;ure njh

$$\frac{\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2-b_2c_1)^2+(c_1a_2-c_2a_1)^2+(a_1b_2-a_2b_1)^2}} \text{ gA}$$

- nk lekrj j[kkvk $\vec{r}=\vec{a}_1+\lambda\vec{b}$ vkj $\vec{r}=\vec{a}_2+\mu\vec{b}$ oQ chp dh njh

$$\left|\frac{\vec{b}\times(\vec{a}_2-\vec{a}_1)}{|\vec{b}|}\right| \text{ gA}$$

- ,d lery] ftldh ey ¯cn l njh d rFkk lery ij ey ¯cn l vfHkyc bdkb lfn'k $\hat{n}$ g] dk lfn'k :i e lehdj.k $\vec{r}\cdot\hat{n}=d$ gA

- ,d lery] ftldh ey ¯cn l njh d rFkk lery oQ vfHkyc dh fnoQ&dkslkbu l, m, n g] dk lehdj.k $lx+my+nz=d$ gA

- ,d ¯cn ftldk fLFkfr lfn'k $\vec{a}$ l tku okyk vkj lfn'k $\vec{N}$ ij yc lery dk lehdj.k $(\vec{r}-\vec{a})\cdot\vec{N}=0$ gA

- ,d fn, x, ¯cn (x_1, y_1, z_1) tku oky vkj ,d nh xbZ j[kk ftlo fno&vuikr A, B, C g] ij yc lery dk lehdj.k $A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$ gA
- rhu vlj[k ¯cnvk (x_1, y_1, z_1), (x_2, y_2, z_2) vkj (x_3, y_3, z_3) l tku oky lery dk lehdj.k g%

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

- rhu ¯cnvk ftuo fLFkfr lfn'k $\vec{a}, \vec{b}$ vkj $\vec{c}$ dk vrfo"V dju oky lery dk lfn'k lehdj.k $(\vec{r} - \vec{a}) \cdot [(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})] = 0$
- ,d lery tk fun'kk{kk dk $(a, 0, 0)$, $(0, b, 0)$ vkj $(0, 0, c)$ ij dkVrk g] dk lehdj.k $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$ gA
- lerykа $\vec{r} \cdot \vec{n}_1 = d_1$ vkj $\vec{r} \cdot \vec{n}_2 = d_2$ o ifrPNnu l xtju oky lery dk lfn'k lehdj.k $\vec{r} \cdot (\vec{n}_1 + \lambda \vec{n}_2) = d_1 + \lambda d_2$ g, tgk λ ,d ikpy gA
- leryk
  $A_1 x + B_1 y + C_1 z + D_1 = 0$
  vkj $A_2 x + B_2 y + C_2 z + D_2 = 0$
  o ifrPNnu l xtju oky lery dk lehdj.k
  $(A_1 x + B_1 y + C_1 z + D_1) + \lambda(A_2 x + B_2 y + C_2 z + D_2) = 0$ gA
- nk j[kk, $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ vkj $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ lg&ryh; g ;fn

$$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 0$$

- ;fn mijkDr j[kk, fcnvk $A(x_1, y_1, z_1)$ rFkk $B(x_2, y_2, z_2)$ l x

g ;fn $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

- nk ry ftlo lfn'k :i $\vec{r} \cdot \vec{n}_1 = d_1$ vkj $\vec{r} \cdot \vec{n}_2 = d_2$ g rFkk buo chp dk U;u dk.k θ g rc $\theta = \cos^{-1} \dfrac{|\vec{n}_1 \cdot \vec{n}_2|}{|\vec{n}_1||\vec{n}_2|}$

- j[kk $\vec{r} = \vec{a} + \lambda \vec{b}$ vkj ry $\vec{r} \cdot \hat{n} = d$ oQ chp dk U;u dk.k φ gS rc

$$\sin \phi = \left| \frac{\vec{b} \cdot \hat{n}}{|\vec{b}|\,|\hat{n}|} \right|$$

- ryk $A_1 x + B_1 y + C_1 z + D_1 = 0$ rFkk $A_2 x + B_2 y + C_2 z + D_2 = 0$ oQ chp dk U;u dk.k θ gS rc

$$\theta = \cos^{-1} \left| \frac{A_1 A_2 + B_1 B_2 + C_1 C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2}\,\sqrt{A_2^2 + B_2^2 + C_2^2}} \right|$$

- lfn'k :i esa ,d fcnq ftldk fLFkfr lfn'k $\vec{a}$ gS ls ry $\vec{r} \cdot \hat{n} = d$ ls nwjh $|d - \vec{a} \cdot \hat{n}|$ gSA

- ,d fcnq (x_1, y_1, z_1) dh ry $Ax + By + Cz + D = 0$ ls nwjh

$$\left| \frac{Ax_1 + By_1 + Cz_1 + D}{\sqrt{A^2 + B^2 + C^2}} \right| \text{ gSA}$$