Set-II

DESIGN OF THE QUESTION PAPER

MATHEMATICS – CLASS XII

Time : 3 Hours
Max. Marks : 100

The weightage of marks over different dimensions of the question paper shall be as follows:

(A) Weightage to different topics/content units

S.No.	Topic	Marks
1.	Relations and functions	10
2.	Algebra	13
3.	Calculus	44
4.	Vectors and three-dimensional geometry	17
5.	Linear programming	06
6.	Probability	10
	Total	**100**

(B) Weightage to different forms of questions:

S.No.	Form of Questions	Marks for each Question	Total No. of Questions	Total Marks
1.	MCQ/Objective type/VSA	01	10	10
2.	Short Answer Questions	04	12	48
3.	Long Answer Questions	06	07	42
	Total		**29**	**100**

(C) Scheme of Option

There is no overall choice. However, an internal choice in four questions of four marks each and two questions of six marks each has been provided.

Blue Print

Units/Type of Question	MCQ/VSA	S.A.	L.A.	Total
Relations and functions	2(2)	8 (2)	–	10 (4)
Algebra	3 (3)	4 (1)	6 (1)	13 (5)
Calculus	2 (2)	24(6)	18(3)	44 (11)
Vectors and 3-dimensional geometry	3 (3)	8 (2)	6 (1)	17 (6)
Linear programming	–	–	6 (1)	6 (1)
Probability	–	4 (1)	6 (1)	10 (2)
Total	**10 (10)**	**48 (12)**	**42 (7)**	**100 (29)**

Section–A

Choose the correct answer from the given four options in each of the Questions 1 to 3.

1. If $*$ is a binary operation given by $*: \mathbf{R} \times \mathbf{R} \rightarrow \mathbf{R}$, $a * b = a + b^2$, then $-2*5$ is

(A) –52 (B) 23 (C) 64 (D) 13

2. If $\sin^{-1} : [-1, 1] \rightarrow \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ is a function, then value of $\sin^{-1}\left(-\frac{1}{2}\right)$ is

(A) $\frac{-\pi}{6}$ (B) $\frac{-\pi}{6}$ (C) $\frac{5\pi}{6}$ (D) $\frac{7\pi}{6}$

3. Given that $\begin{pmatrix} 9 & 6 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 3 & 0 \\ 1 & 2 \end{pmatrix}$. Applying elementary row transformation $R_1 \rightarrow R_1 - 2R_2$ on both sides, we get

(A) $\begin{pmatrix} 3 & 6 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & -4 \\ 1 & 2 \end{pmatrix}$ (B) $\begin{pmatrix} 3 & 6 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 3 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 3 & 0 \\ 1 & 2 \end{pmatrix}$

(C) $\begin{pmatrix} -3 & 6 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 3 & 0 \\ -3 & 2 \end{pmatrix}$ (D) $\begin{pmatrix} -3 & 6 \\ 3 & 0 \end{pmatrix} = \begin{pmatrix} -4 & 3 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 3 & 0 \\ 1 & 2 \end{pmatrix}$

4. If A is a square matrix of order 3 and $|A| = 5$, then what is the value of $|\text{Adj. } A|$?

5. If A and B are square matrices of order 3 such that $|A| = -1$ and $|B| = 4$, then what is the value of $|3(AB)|$?

6. The degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^3\right] = \left(\frac{d^2y}{dx^2}\right)^2$ is_______.

Fill in the blanks in each of the Questions 7 and 8:

7. The integrating factor for solving the linear differential equation $x\frac{dy}{dx} - y = x^2$ is_______.

8. The value of $\left|\hat{i}-\hat{j}\right|^2$ is________.

9. What is the distance between the planes $3x + 4y - 7 = 0$ and $6x + 8y + 6 = 0$?

10. If $\vec{a}$ is a unit vector and $(\vec{x} - \vec{a}).(\vec{x} + \vec{a}) = 99$, then what is the value of $|\vec{x}|$?

Section–B

11. Let n be a fixed positive integer and R be the relation in **Z** defined as a R b if and only if $a - b$ is divisible by n, $\forall\ a, b \in Z$. Show that R is an equivalence relation.

12. Prove that $\cot^{-1}7 + \cot^{-1}8 + \cot^{-1}18 = \cot^{-1}3$.

OR

Solve the equation $\tan^{-1}(2+x)+\tan^{-1}(2-x)=\tan^{-1}\frac{2}{3}, -\sqrt{3}>x>\sqrt{3}$.

13. Solve for x, $\begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix} = 0$

OR

If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 2 & -3 \end{pmatrix}$, verify that $(AB)' = B'A'$.

14. Determine the value of k so that the function:

$$f(x) = \begin{cases} \dfrac{k.\cos 2x}{\pi - 4x}, & \text{if } x \neq \dfrac{\pi}{4} \\ 5, & \text{if } x = \dfrac{\pi}{4} \end{cases}$$

is continuous at $x = \frac{\pi}{4}$.

15. If $y = e^{a\cos^{-1}x}$, show that $(1 - x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - a^2y = 0$.

16. Find the equation of the tangent to the curve $x = \sin 3t$, $y = \cos 2t$ at $t = \dfrac{\pi}{4}$.

Find the intervals in which the function $f(x) = \sin^4 x + \cos^4 x$, $0 < x < \dfrac{\pi}{2}$, is strictly increasing or strictly decreasing.

17. Evaluate $\displaystyle\int_0^{\frac{\pi}{6}} \sin^4 x \cos^3 x \, dx$

18. Evaluate $\displaystyle\int \frac{3x+1}{2x^2-2x+3}\,dx$

OR

Evaluate $\displaystyle\int x.(\log x)^2\,dx$

19. Find a particular solution of the differential equation

$2y\, e^{\frac{x}{y}}\, dx + (y - 2x e^{\frac{x}{y}})\, dy = 0$, given that $x = 0$ when $y = 1$.

20. If $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$, then find the projection of $\vec{b} + \vec{c}$ along $\vec{a}$.

21. Determine the vector equation of a line passing through (1, 2, –4) and perpendicular to the two lines $\vec{r} = (8\hat{i} - 16\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$ and $(15\hat{i} + 29\hat{j} + 5\hat{k}) + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$.

22. There are three coins. One is a biased coin that comes up with tail 60% of the times, the second is also a biased coin that comes up heads 75% of the times and the third is an unbiased coin. One of the three coins is chosen at random and tossed, it showed heads. What is the probability that it was the unbiased coin?

SECTION–C

23. Find A^{-1}, where $A = \begin{pmatrix} 4 & 1 & 3 \\ 2 & 1 & 1 \\ 3 & 1 & -2 \end{pmatrix}$. Hence solve the following system of equations $4x + 2y + 3z = 2,\ x + y + z = 1,\ 3x + y - 2z = 5,$

OR

Using elementary transformations, find A^{-1}, where

$$A = \begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$$

24. Show that the semi-vertical angle of the cone of maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.

25. Evaluate $\int_1^3 (3x^2 + 2x + 5)\, dx$ by the method of limit of sum.

26. Find the area of the triangle formed by positive x-axis, and the normal and tangent to the circle $x^2 + y^2 = 4$ at $(1, \sqrt{3})$, using integration.

27. Find the equation of the plane through the intersection of the planes $x + 3y + 6 = 0$ and $3x - y - 4z = 0$ and whose perpendicular distance from origin is unity.

OR

Find the distance of the point (3, 4, 5) from the plane $x + y + z = 2$ measured parallel to the line $2x = y = z$.

28. Four defective bulbs are accidently mixed with six good ones. If it is not possible to just look at a bulb and tell whether or not it is defective, find the probability distribution of the number of defective bulbs, if four bulbs are drawn at random from this lot.

29. A furniture firm manufactures chairs and tables, each requiring the use of three machines A, B and C. Production of one chair requires 2 hours on machine A, 1 hour on machine B and 1 hour on machine C. Each table requires 1 hour each on machine A and B and 3 hours on machine C. The profit obtained by selling one chair is Rs 30 while by selling one table the profit is Rs 60. The total time available per week on machine A is 70 hours, on machine B is 40 hours and on machine C is 90 hours. How many chairs and tables should be made per week so as to maximise profit? Formulate the problems as a L.P.P. and solve it graphically.

Marking Scheme

Section-A

1. (B)	**2.** (D)	**3.** (B)		
4. 25	**5.** –108	**6.** 2	**7.** $\frac{1}{x}$	**Marks**
8. 2	**9.** 2 Units	**10.** 10		$1 \times 10 = 10$

Sections-B

11. (i) Since a R a, $\forall\, a \in$ Z, and because 0 is divisible by n, therefore R is reflexive. 1

(ii) a R $b \Rightarrow a - b$ is divisible by n, then $b - a$, is divisible by n, so b R a. Hence R is symmetric. 1

(iii) Let a R b and b R c, for $a,b,c, \in$ **Z**. Then $a - b = n\,p$, $b - c = n\,q$, for some $p, q \in$ **Z**

Therefore, $a - c = n\,(p + q)$ and so a R c. 1

Hence R is reflexive and so equivalence relation. 1

12. LHS $= \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8} + \tan^{-1}\frac{1}{18}$ 1

$$= \tan^{-1}\frac{\frac{1}{7}+\frac{1}{8}}{1-\frac{1}{7}.\frac{1}{8}} + \tan^{-1}\frac{1}{18} = \tan^{-1}\left(\frac{15}{55}\right) + \tan^{-1}\frac{1}{18}$$ 1

$$= \tan^{-1}\frac{3}{11} + \tan^{-1}\frac{1}{18} = \tan^{-1}\frac{\frac{3}{11}+\frac{1}{18}}{1-\frac{3}{11}\frac{1}{18}} = \tan^{-1}\frac{65}{195}$$ 1

$$= \tan^{-1}\frac{1}{3} = \cot^{-1}3 = \text{RHS}$$ 1

OR

Since $\tan^{-1}(2+x) + \tan^{-1}(2-x) = \tan^{-1}\frac{2}{3}$

Therefore, $\tan^{-1}\frac{(2+x)+(2-x)}{1-(2+x)(2-x)} = \tan^{-1}\frac{2}{3}$ 1½

Thus $\frac{4}{x^2-3} = \frac{2}{3}$ 1½

$\Rightarrow \quad x^2 = 9 \Rightarrow x = \pm 3$ 1

13. Given, $\begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix} = 0$

Using $\begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$, we get $\begin{vmatrix} x+2 & x+6 & x-1 \\ 4 & -7 & 3 \\ -3 & -4 & 7 \end{vmatrix} = 0$ 1½

Using $\begin{matrix} C_2 \to C_2 - C_1 \\ C_3 \to C_3 - C_1 \end{matrix}$, we get $\begin{vmatrix} x+2 & 4 & -3 \\ 4 & -11 & -1 \\ -3 & -1 & 10 \end{vmatrix} = 0$ 1½

Therefore, $(x+2)(-111) - 4(37) - 3(-37) = 0$

which on solving gives $x = -\frac{7}{3}$ 1

OR

$AB = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & -1 & 2 \\ 3 & 2 & -3 \end{pmatrix} = \begin{pmatrix} 7 & 3 & -4 \\ 15 & 5 & -6 \end{pmatrix}$ 1

Therefore, LHS = (AB)′ = $\begin{pmatrix} 7 & 15 \\ 3 & 5 \\ -4 & -6 \end{pmatrix}$ 1

RHS = B′ A′ = $\begin{pmatrix} 1 & 3 \\ -1 & 2 \\ 2 & -3 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 7 & 15 \\ 3 & 5 \\ -4 & -6 \end{pmatrix}$ and hence LHS = RHS

1+1

14. Since f is continous at $x = \frac{\pi}{4}$, we have $\lim_{x \to \frac{\pi}{4}} f(x) = 5$.

Now $\lim_{x \to \frac{\pi}{4}} f(x) = \lim_{x \to \frac{\pi}{4}} \frac{k.\cos 2x}{\pi - 4x} = \lim_{y \to 0} \frac{k\cos 2(\frac{\pi}{4} - y)}{\pi - 4(\frac{\pi}{4} - y)}$, where $\frac{\pi}{4} - x = y$, 1

$= \lim_{y \to 0} \frac{k.\cos(\frac{\pi}{2} - 2y)}{\pi - \pi + 4y} = \lim_{y \to 0} \frac{(k\sin 2y)}{2.\,2y} = \frac{k}{2}$ 1

Therefore, $\frac{k}{2} = 5 \Rightarrow k = 10.$ 1

15. $y = e^{a\cos^{-1}x} \Rightarrow \frac{dy}{dx} = e^{a\cos^{-1}x}\frac{(-a)}{\sqrt{1-x^2}}$ ½

Therefore, $\sqrt{1-x^2}\,\frac{dy}{dx} = -a\,y$.......(1) ½

Differentiating again w.r.t. x, we get

$$\sqrt{1-x^2}\,\frac{d^2y}{dx^2} - \frac{x}{\sqrt{1-x^2}}\frac{dy}{dx} = -\frac{ady}{dx}$$ 1½

$$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = -a\sqrt{1-x^2}\,\frac{dy}{dx}$$ ½

$= -\, a\,(-\, ay)$ [from 1] ½

Hence $(1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - a^2y = 0.$ ½

16. $\dfrac{dx}{dt} = +\,3\cos 3t,\ \dfrac{dy}{dt} = -\,2\sin 2t$ 1

Therefore, $\dfrac{dy}{dx} = -\dfrac{2\sin 2t}{3\cos 3t}$, and $\left(\dfrac{dy}{dx}\right)_{t=\frac{\pi}{4}} = \dfrac{-2\sin\frac{\pi}{2}}{3\cos 3\frac{\pi}{4}} = \dfrac{-2}{3.(-\frac{1}{\sqrt{2}})} = \dfrac{2\sqrt{2}}{3}$ 1

Also $x = \sin 3t = \sin 3\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}}$ and $y = \cos 2t = \cos\dfrac{\pi}{2} = 0.$

Therefore, Point is $\left(\dfrac{1}{\sqrt{2}}, 0\right)$ 1

Hence, equation of tangent is $y - 0 = \dfrac{2\sqrt{2}}{3}\left(x - \dfrac{1}{\sqrt{2}}\right)$

$2\sqrt{2}\,x - 3y - 2 = 0$ 1

OR

$f'(x) = 4\sin^3 x \cos x - 4\cos^3 x \sin x$

$= -4\sin x \cos x(\cos^2 x - \sin^2 x)$

$= -\sin 4x$. Therefore, 1

$f'(x) = 0 \Rightarrow 4x = n\pi \Rightarrow x = n\frac{\pi}{4}$

Now, for $0 < x < \frac{\pi}{4}$, 1

$f'(x) < 0$

Therefore, f is strictly decreasing in $(0, \frac{\pi}{4})$ 1½

Similarly, we can show that f is strictly increasing in $(\frac{\pi}{4}, \frac{\pi}{2})$ ½

17. $I = \int_0^{\frac{\pi}{6}} \sin^4 x \cos^3 x \, dx$

$= \int_0^{\frac{\pi}{6}} \sin^4 x(1 - \sin^2 x)\cos x \, dx$ 1

$= \int_0^{\frac{1}{2}} t^4(1 - t^2)\, dt$, where $\sin x = t$ 1

$= \int_0^{\frac{1}{2}} (t^4 - t^6)\, dt = \left[\frac{t^5}{5} - \frac{t^7}{7}\right]_0^{\frac{1}{2}}$ 1

$= \frac{1}{5}\left(\frac{1}{2}\right)^5 - \frac{1}{7}\left(\frac{1}{2}\right)^7 = \frac{1}{32}\left(\frac{1}{5} - \frac{1}{28}\right) = \frac{23}{4480}$ 1

18. $\text{I} = \int \frac{3x+1}{2x^2-2x+3}dx = \int \frac{\frac{3}{4}(4x-2)+\frac{5}{2}}{2x^2-2x+3}dx$ 1

$$= \frac{3}{4}\int \frac{4x-2}{2x^2-2x+3}dx + \frac{5}{4}\int \frac{1}{x^2-x+\frac{3}{2}}dx$$

$$= \frac{3}{4}\log|2x^2-2x+3| + \frac{5}{4}\int \frac{dx}{\left(x-\frac{1}{2}\right)^2+\left(\frac{\sqrt{5}}{2}\right)^2}$$ $1\frac{1}{2}$

$$= \frac{3}{4}\log|2x^2-2x+3| + \frac{5}{4}\,\frac{2}{\sqrt{5}}\tan^{-1}\frac{2x-1}{\sqrt{5}}+c$$ $1\frac{1}{2}$

$$= \frac{3}{4}\log|2x^2-2x+3| + \frac{\sqrt{5}}{2}\tan^{-1}\frac{2x-1}{\sqrt{5}}+c$$

OR

$$\text{I} = \int x(\log x)^2.dx = \int (\log x)^2\,x\,dx$$

$$= (\log x)^2\frac{x^2}{2} - \int 2\log x\frac{1}{x}\,\frac{x^2}{2}dx$$ 1

$$= \frac{x^2}{2}(\log x)^2 - \int \log x.\,x\;dx$$ $\frac{1}{2}$

$$= \frac{x^2}{2}(\log x)^2 - \left[\log x.\,\frac{x^2}{2} - \int \frac{1}{x}.\frac{x^2}{2}\,dx\right]$$ $1\frac{1}{2}$

$$= \frac{x^2}{2}(\log x)^2 - \frac{x^2}{2}\log x + \frac{x^2}{4} + c$$ 1

19. Given differential equation can be written as

$$\frac{dx}{dy} = \frac{2xe^{\frac{x}{y}} - y}{2y.e^{\frac{x}{y}}}$$ $\frac{1}{2}$

Putting $\frac{x}{y} = v \Rightarrow x = vy \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$ $\frac{1}{2}$

Therefore, $v + y\frac{dv}{dy} = \frac{2vye^v - y}{2ye^v} = \frac{2ve^v - 1}{2e^v}$ $\frac{1}{2}$

$$y\frac{dv}{dy} = \frac{2ve^v - 1}{2e^v} - v$$ 1

Hence $2e^v dv = -\frac{dy}{y}$

$$\Rightarrow 2e^v = -\log|y| + c$$ 1

or $2e^{\frac{x}{y}} = -\log|y| + c$

when $x = 0$, $y = 1$

$\Rightarrow C = 2$

Therefore, the particular solution is $2e^{\frac{x}{y}} = -\log|y| + 2$ $\frac{1}{2}$

20. $\vec{b}+\vec{c}=(\hat{i}+2\hat{j}-3\hat{k})+(2\hat{i}-\hat{j}+4\hat{k})=3\hat{i}+\hat{j}+\hat{k}$ 1

$\vec{a}=2\hat{i}-2\hat{j}+\hat{k}$

Projection of $(\vec{b}+\vec{c})$ along $\vec{a}=\dfrac{(\vec{b}+\vec{c}).\vec{a}}{|\vec{a}|}$ is 1

$\dfrac{6-2+1}{\sqrt{4+4+1}}=\dfrac{5}{3}$ units 1+1

21. A vector perpendicular to the two lines is given as

$$(3\hat{i}-16\hat{j}+7\hat{k})\times(3\hat{i}+8\hat{j}-5\hat{k})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -16 & 7\\ 3 & 8 & -5\end{vmatrix}$$ $1\frac{1}{2}$

$=24\hat{i}+36\hat{j}+72\hat{k}$ or $12(2\hat{i}+3\hat{j}+6\hat{k})$ 1

Therefore, Equation of required line is

$$\vec{r}=(\hat{i}+2\hat{j}-4\hat{k})+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$$ $1\frac{1}{2}$

22. Let E_1: selection of first (biased) coin

E_2: selection of second (biased) coin

E_3: selection of third (unbiased) coin

$P(E_1)=P(E_2)=P(E_3)=\dfrac{1}{3}$ $\frac{1}{2}$

Let A denote the event of getting a head

Therefore, $P\left(\frac{A}{E_1}\right) = \frac{40}{100}$, $P\left(\frac{A}{E_2}\right) = \frac{75}{100}$, $P\left(\frac{A}{E_3}\right) = \frac{1}{2}$ $1\frac{1}{2}$

$$P\left(\frac{E_3}{A}\right) = \frac{P(E_3)P\left(\frac{A}{E_3}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)}$$ $\frac{1}{2}$

$$= \frac{\frac{1}{3}\cdot\frac{1}{2}}{\frac{1}{3}\cdot\frac{40}{100} + \frac{1}{3}\cdot\frac{75}{100} + \frac{1}{3}\cdot\frac{1}{2}} = \frac{10}{33}$$ $1\frac{1}{2}$

SECTION–C

23. $|A| = 4(-3) - 1(-7) + 3(-1) = -12 + 7 - 3 = -8$ 1

$A_{11} = -3 \quad A_{12} = 7 \quad A_{13} = -1$ $1\frac{1}{2}$

$A_{21} = 5 \quad A_{22} = -17 \quad A_{23} = -1$

$A_{31} = -2 \quad A_{32} = 2 \quad A_{33} = 2$

Therefore, $A^{-1} = -\frac{1}{8}\begin{pmatrix} -3 & 5 & -2 \\ 7 & -17 & 2 \\ -1 & -1 & 2 \end{pmatrix}$ $\frac{1}{2}$

Given equations can be written as

$$\begin{pmatrix} 4 & 2 & 3 \\ 1 & 1 & 1 \\ 3 & 1 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$$

$\Rightarrow A'.X = B \Rightarrow X = (A'^{-1})B$ 1

$= (A^{-1})'B$

$$\Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{-1}{8} \begin{pmatrix} -3 & 7 & -1 \\ 5 & -17 & -1 \\ -2 & 2 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}$$

$$= -\frac{1}{8} \begin{pmatrix} -6 & +7 & -5 = & -4 \\ 10 & -17 & -5 = & -12 \\ -4 & +2 & +10 = & 8 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{3}{2} \\ -1 \end{pmatrix}$$ $1\frac{1}{2}$

Therefore, $x = \frac{1}{2},\ y = \frac{3}{2},\ z = -1$ $\frac{1}{2}$

OR

Writing $A = \begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$ $\frac{1}{2}$

$R_2 \to R_2 + R_1 \Rightarrow \begin{pmatrix} 1 & 2 & -2 \\ 0 & 5 & -2 \\ 0 & -2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$ 1

$R_2 \to R_2 + 2R_3 \Rightarrow \begin{pmatrix} 1 & 2 & -2 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} A$ 1

$$R_3 \to R_3 + 2R_2 \Rightarrow \begin{pmatrix} 1 & 2 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{pmatrix}$$ 1

$$R_1 \to R_1 + 2R_3 \Rightarrow \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 4 & 10 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{pmatrix} A$$ 1

$$R_1 \to R_1 - 2R_2 \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{pmatrix} A$$ 1

$$\Rightarrow A^{-1} = \begin{pmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{pmatrix}$$ $\frac{1}{2}$

24. Volume $v = v = \frac{1}{3}\pi r^2 h$ $\frac{1}{2}$

$l^2 = h^2 + r^2$ $1\frac{1}{2}$

$v = \frac{1}{3}\pi\ (l^2 - h^2)\ h = \frac{1}{3}\pi\ (l^2 h - h^3)$ $\frac{1}{2}$

$$\frac{dv}{dh} = \frac{\pi}{3}(l^2 - 3h^2) = 0$$ $1\frac{1}{2}$ 1

$l = \sqrt{3}h,\quad r = \sqrt{2}h$

$\tan\alpha = \frac{r}{h} = \sqrt{2}$

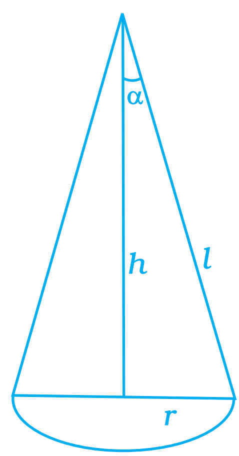


Fig. 2.1

$\alpha = \tan^{-1}\sqrt{2}$

$\frac{d^2v}{dh^2} = -2\pi h < 0$ 1

Therefore, v is maximum

25. $I = \int_1^3 (3x^2 + 2x + 5)\,dx = \int_1^3 f(x)\,dx$

$= \lim_{h \to o} h\left[f(1) + f(1+h) + f(1+2h) + ... + f(1+(n-1)h)\right]$ ...... (i) 1

where $h = \frac{3-1}{n} = \frac{2}{n}$

Now

$f(1) = 3 + 2 + 5 = 10$

$f(1+h) = 3 + 3h^2 + 6h + 2 + 2h + 5 = 10 + 8h + 3h^2$

$f(1+2h) = 3 + 12h^2 + 12h + 2 + 4h + 5 = 10 + 8.2.h + 3.2^2.h^2$ $1\frac{1}{2}$

$f(1+(n-1)h) = 10 + 8(n-1)\,h + 3(n-1)^2.h^2$

$I = \lim_{n \to \infty} h\left[10n + 8h\frac{n(n-1)}{2} + 3h^2\frac{n(n-1)(2n-1)}{6}\right]$ $1\frac{1}{2}$

$= \lim_{n \to \infty} \frac{2}{n}\left[10n + \frac{16}{n}\frac{n(n-1)}{2} + \frac{12}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$

$$= \lim_{n\to\infty} \frac{2}{n}\left[10n + 8(n-1)\frac{2}{n}(n-1)\,(2n-1)\right]$$ $\frac{1}{2}$

$$= \lim_{n\to\infty} 2\left[10 + 8(1-\frac{1}{n}) + 2(1-\frac{1}{n})\,(2-\frac{1}{n})\right]$$ 1

$$= 2\,[10 + 8 + 4] = 44$$ $\frac{1}{2}$

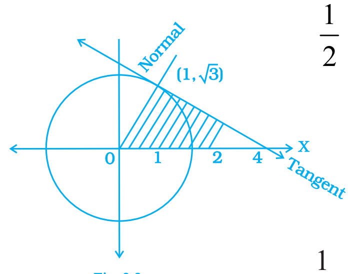


Fig. 2.2

26. Equation of tangent to $x^2 + y^2 = 4$ at $(1, \sqrt{3})$ is

$x + \sqrt{3}y = 4$. Therefore, $y = \frac{4-x}{\sqrt{3}}$ 1

Equation of normal $y = \sqrt{3}x$ 1

Therefore, required area $= \int_0^1 \sqrt{3}\,x\,dx + \int_1^4 \frac{4-x}{\sqrt{3}}\,dx$ 1

$$= \left(\sqrt{3}\,\frac{x^2}{2}\right)_0^1 + \frac{1}{\sqrt{3}}\left(4x - \frac{x^2}{2}\right)_1^4$$ 1

$$= \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}}\left[8 - \frac{7}{2}\right] = \frac{\sqrt{3}}{2} + \frac{3\sqrt{3}}{2} = 2\sqrt{3} \text{ sq. units}$$ 2

27. Equation of required plane is

$(x + 3y + 6) + \lambda\,(3x - y - 4z) = 0$ $1\frac{1}{2}$

$\Rightarrow$ $(1 + 3\,\lambda)\,x + (3 - \lambda)\,y - 4\,\lambda\,z + 6 = 0$ $\frac{1}{2}$

Perpendicular distance to the plane from origin is

Therefore, $\frac{6}{\sqrt{(1+3\lambda)^2+(3-\lambda)^2+(-4\lambda)^2}}=1$ $1\frac{1}{2}$

or $\quad 36 = 1+ 9\lambda^2 + 6\lambda + 9 + \lambda^2 - 6\lambda + 16\lambda^2$

or $\quad 26\lambda^2 = 26 \Rightarrow \lambda = \pm 1$

Equations of required planes are

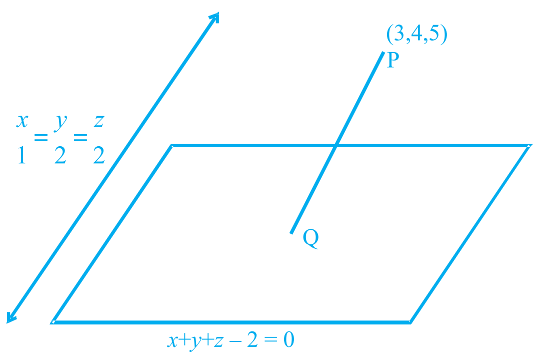
$4x + 2y - 4z + 6 = 0$ and $-2x + 4y + 4z + 6 = 0$ $1\frac{1}{2}$

or $2x + y - 2z + 3 = 0$ and $x - 2y - 2z - 3 = 0$ 1

OR

Equaiton of line is $2x = y = z$ i.e. $\frac{x}{\frac{1}{2}} = \frac{y}{1} = \frac{z}{1}$

or $\quad \frac{x}{1} = \frac{y}{2} = \frac{z}{2}$ 1



1

Fig. 2.3

Equation of line P Q is

$$\frac{x-3}{1}=\frac{y-4}{2}=\frac{z-5}{2}=\lambda \qquad 1$$

$\Rightarrow Q(\lambda+3,\ 2\lambda+4, 2\lambda+5)$ lies on plane. Therefore,

$$\lambda+3+2\lambda+4+2\lambda+5-2=0 \qquad 1\frac{1}{2}$$

or $5\lambda=-10$ gives $\lambda=-2$ which gives the coordinates of $Q(1, 0, 1)$

Therefore, $PQ=\sqrt{4+16+16}=$ 6 units $\qquad 1\frac{1}{2}$

28. Let x denotes the number of defective bulbs

$$P(X=0)=\frac{{}^{6}C_{4}}{{}^{10}C_{4}}=\frac{6.5.4.3}{10.9.8.7}=\frac{1}{14} \qquad 1$$

$$P(X=1)=\frac{{}^{6}C_{3}\,{}^{4}C_{1}}{{}^{10}C_{4}}=\frac{6.5.4.4.}{10.9.8.7}4=\frac{8}{21} \qquad 1$$

$$P(X=2)=\frac{{}^{6}C_{2}\,{}^{6}C_{2}}{{}^{10}C_{4}}=\frac{6.5.4.3}{10.9.8.7}.6=\frac{3}{7} \qquad 1$$

$$P(X=3)=\frac{{}^{6}C_{1}\,{}^{6}C_{3}}{{}^{10}C_{4}}=\frac{6.4.3.2}{10.9.8.7}.4=\frac{4}{35} \qquad 1$$

$$P(X=4)=\frac{{}^{4}C_{4}}{{}^{10}C_{4}}=\frac{4.3.2.1}{10.9.8.7}=\frac{1}{210} \qquad 1$$

Therefore, distribution is

X :	0	1	2	3	4
P (X) :	$\frac{1}{14}$	$\frac{8}{21}$	$\frac{3}{7}$	$\frac{4}{35}$	$\frac{1}{210}$

1

29. Let number of chairs to be made per week be x and tables be y

Thus we have to maximise $P = 30\,x + 60\,y$

Subject to $2\,x + y \le 70$

$x + y \le 40$ 2

$x + 3y \le 90$

$x \ge 0 y \ge 0$

Vertices of feasible region are 2

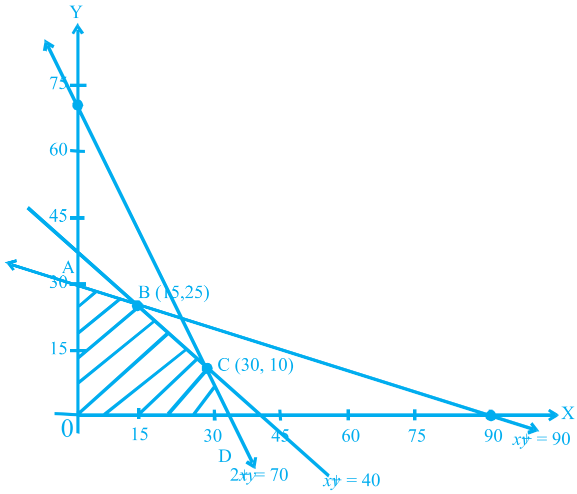


Fig. 2.4

A (0,30), B (15, 25), C (30,10), D (35, 0) $\frac{1}{2}$

P (at A) = 30 (60) = 1800

P (at B) = 30 (15 + 50) = 1950

$$P \text{ (at C)} = 30\,(30 + 20) = 1500 \qquad 1\frac{1}{2}$$

$$P \text{ (at D)} = 30\,(35) = 1050$$

P is Maximum for 15 chairs and 25 tables.