

Sample Paper- 2015
Subject: Mathematics
Class 12th

Time allowed: 3 hours

Maximum Marks: 100

General instructions:

- a. All questions are compulsory.
- b. Question number 1 to 10 is of 1 mark each. Question numbers 11 to 22 are of 4 marks each. Question numbers 23 to 29 are of 6 marks each.
- c. Internal choices have been provided in some questions. You have to attempt only one of the choices in such questions.
- d. Use of calculator is not permitted

1. Find the matrix X , if $X + \begin{bmatrix} 2 & 6 \\ -3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 3 & -5 \end{bmatrix}$
2. If $f(x)$ is an invertible function, find the inverse of $f(x) = \frac{3x-2}{5}$
3. Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3 x \, dx$
4. Evaluate: $\int \sin^{-1}(\cos x) \, dx$
5. Evaluate: $\sin\left\{\frac{\pi}{2} - \sin^{-1}\frac{-\sqrt{3}}{2}\right\}$
6. Find the cofactor of a_{21} in the $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$
7. If $A = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$, find the inverse of A .
8. Find the area of the parallelogram whose adjacent sides are the vectors $3\hat{i} + \hat{j} - 2\hat{k}$ and $\hat{i} - 3\hat{j} + 4\hat{k}$.
9. Find the equation of the plane which is parallel to the plane $2x - y + 2z - 5 = 0$ which passes through the point $(1, 1, 1)$.
10. Find the length of the perpendicular from the point $2\hat{i} + \hat{j} - \hat{k}$ to the plane $r \cdot (\hat{i} - 2\hat{j} + 4\hat{k}) = 9$.
11. Show that the equation of normal to the curve $x^2 = 4y$ which passes through the point $(1, 2)$.
12. Find the interval on which the function $f(x) = 2x^3 - 15x^2 + 36x + 6$ is increasing or decreasing

OR

Differentiate w.r.to x : $\tan^{-1} \left\{ \sqrt{\frac{1+\sin x}{1-\sin x}} \right\}$

13. Show that the relation '*' on Z defined by $a * b = a + b + 1$ for all $a, b \in Z$, satisfies closure property. Also find the identity element in Z . What is the inverse of an element $a \in Z$?
14. Evaluate: $\int \frac{2x}{(x^2+2)(x^2+3)} \, dx$

OR

$$\int \frac{1}{x^2 + 8x + 4} dx$$

15. If $y = \tan^{-1} \sqrt{x^2 - 1}$, then show that $x(x^2 - 1)y_2 - (2x^2 - 1)y_1 = 0$

16. A bag X contains 2 white and 3 red balls and a bag Y contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag Y?

OR

Three persons A, B and C throw a coin alternatively till one of them gets a 'head' and wins the game. Find their respective probabilities of winning, if A starts the game followed by B and C.

17. Using the properties of determinant prove that $\begin{vmatrix} a+b+2c & c & c \\ a & 2a+b+c & a \\ b & b & a+2b+c \end{vmatrix} = 2(a+b+c)^2$.

18. Solve the differential equation $(x^2 + 3y^2)dx = 2xydy$.

19. Solve the differential equation $(1 - x^2) \frac{dy}{dx} + 2xy = \sqrt{1 - x^2}$.

20. Find the equation of the plane passing through the point P (-1, -1, 2) and Q (2, -2, 2) perpendicular to the plane $x+2y+2z=5$.

OR

Find the image of the point (1, 2, 3) in the plane $x+2y+4z=38$.

21. Prove that $\tan^{-1} \frac{1}{4} + \tan^{-1} \frac{2}{9} = \frac{1}{2} \tan^{-1} \frac{4}{3}$

22. Find a vector whose magnitude is 3 units and which is perpendicular to the following vectors and $a=3i^{\wedge}+j^{\wedge}-4k^{\wedge}$ and $b=6i^{\wedge}+5j^{\wedge}-2k^{\wedge}$

23. Given $A = \begin{vmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{vmatrix}$ and $B = \begin{vmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{vmatrix}$, find AB and use this result in solving the system of equations: $x - y + z = 4$, $x - 2y - 2z = 9$, $2x + y + 3z = 1$

24. Find the equation of the plane passing through the point (1, 2, 3) and (0, -1, 0) and parallel to the line $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z}{-3}$.

25. A coin is tossed 4 times. If X is the number of heads observed, find the probability distribution of X . Also find the variance?
26. A dietician wishes to mix together two kinds of food X and Y in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin contents of 1 kg food is given below:

Food	Vitamin A	Vitamin B	Vitamin C
X	1	2	3
Y	2	2	1

One kg of food X costs 16 and one kg of food Y costs 20. Find the least cost of the mixture which will produce the required diet?

27. Show that the semi-vertical angle of a right circular cone of maximum volume and slant height is $\tan^{-1} \sqrt{2}$

OR

An open box with a square base is to be made out of a given quantity of cardboard of area a^2 sq unit. Find the dimensions of the box so that the volume of the box is maximum. Also find the maximum volume.

28. Evaluate: $\int_1^4 (x^2 + 2) dx$ as limit of sums.

OR

Sketch the region and find the area bounded by the circle $x^2 + y^2 = 16$ and the line $y = x$ in the first quadrant..

29. Prove that $\int_0^{\pi} \frac{x \tan x}{\sec x \cos ecx} dx = \frac{\pi^2}{4}$