

SAMPLE PAPER-2015
MATHEMATICS
CLASS XII

Time:3Hrs

M.M:100

General Instructions

- ❖ All questions are compulsory.
- ❖ The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of **one** mark each, section B comprises of 12 questions of **four** marks each and section C comprises of 07 questions of **six** mark each.
- ❖ All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
- ❖ There is no overall choice. However, internal choice has been provided in 04 questions of **four** marks each and 2 questions of **six** marks each. You have to attempt only one of the alternatives in all such questions.
- ❖ Use of calculators in not permitted. You may ask for logarithmic tables, if required

SECTION – A

Question numbers 1 to 10 carry 1 mark each.

1. Let * be a binary operation on N given by $a * b = \text{HCF}(a, b)$ for all $a, b \in \mathbb{N}$. Find $5 * 7$.
2. Find unit vector in the direction of the vector $\vec{a} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$
3. For a 2×2 matrix $A = [a_{ij}]$, Whose elements are given by $a_{ij} = \frac{(i+2j)^w}{4}$, Write a_{22} .
4. Evaluate: $\int \frac{(x+1)(x+\log x)^2}{x} dx$
5. If $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$, Write the minor of the element a_{22} .
6. For what value of p, $\begin{vmatrix} 1+p & 7 \\ 3-x & 8 \end{vmatrix}$ is a singular matrix?
7. Write the distance between two planes: $2x + 3y + 4z = 5$ and $4x + 6y + 8z = 10$.
8. If α, β, γ are the angles which make with the positive direction of axes. Find the value of $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$.
9. Find the principal value of $\tan^{-1} \sqrt{3 - \sec^{-1}(-2)}$.
- 10 Evaluate $\int e^{3 \log x} \cdot x^4 dx$

SECTION – B

Question numbers 11 to 22 carry 4 marks each.

11. Let * be a binary operation on Q Defined by $a * b = \frac{2ab}{5}$. Show that * is commutative as well as associative. Also find it's identify element, If it exists.
12. Solve for x: $\tan^{-1} \frac{1-x}{1+x} = \frac{1}{2} \tan^{-1} x = 0, x > 0$

OR

Prove that $\cos [\tan^{-1} \{ \sin(\cot^{-1} x) \}] = \sqrt{\frac{1+x^2}{2+x^2}}$

13. Find the value of k so that f is continuous at the indicated point

$$f(x) = \begin{cases} \frac{1 - \cos 4x}{x^2} & x < 0 \\ a & x = 0 \\ \frac{\sqrt{x}}{\sqrt{16 + \sqrt{x} - 4}} & x > 0 \end{cases}$$

If f(x) is continuous at x = 0, find the value of a.

14. Find the intervals in which the function f given by $f(x) = 2x^3 - 3x^2 - 36x + 7$ is (a) strictly increasing (b) strictly decreasing

OR

Use differential to approximate $\sqrt{36.6}$

15. Prove that
$$\begin{vmatrix} a & b & y \\ a^2 & b^2 & y^2 \\ B + y & a + y & a + b \end{vmatrix} = (a-b)(b-y)(y-a)(a+b+y)$$

16. If $y = 3\cos(\log x) + 4\sin(\log x)$, Show that: $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$

OR

If $\sin y = x \sin(a + y)$, Prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

17. If the lines $\frac{x-1}{3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ are perpendicular to each other. Find the value of k.

18. The probability of a shooter hitting a target is $\frac{3}{4}$. How many maximum numbers of times must he/she fire so that the probability of hitting the target at least once is more than 0.99?

19. Show that $(a-b) \times (a+b) = 2(a \times b)$

20. Find the general solution of the differential equation $y dx + (x - y^2) dy = 0$

21. Find the particular solution of the differential equation $\frac{dy}{dx} + \frac{y^2 + y + 1}{x^2 + x + 1} = 0$ is given by $(x + y + 1) = A(1 - x - y - 2xy)$, where A is parameter.

22. Evaluate: $\int \frac{1}{\sin x \cos^3 x} dx$ **OR** Evaluate: $\int \frac{1}{x(x^4 - 1)} dx$

Question numbers 23 to 29 carry 6 marks each

23. Find the coordinate of foot of perpendicular drawn from point (1, 6, 3) on the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ and also find the image of the point (1, 6, 3) in the given line.

OR

Find the vector equation of the plane passing through the intersection of planes $r \cdot (2i - 7j + 4k) = 3$ and $r \cdot (3i - 5j + 4k) + 11 = 0$ and passing through the point (-2, 1, 3)

24. If a machine is correctly set up, it produces 90 % acceptable items. If it is incorrectly set up, it produces only 40 % acceptable items. Past experience shows that 80 % of the set ups are correctly done. If after a certain set up, the

machine produces 2 acceptable items, Find the probability that the machine is correctly set up.

25. Find the area of the region $\{(x, y): 0 \leq y \leq x^2 + 1, 0 \leq y \leq x+1, 0 \leq x \leq 2\}$
 $2x + y = 4, 3x - 2y = 6$ and $x - 3y + 5 = 0$

26. Evaluate: $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx$

27. For the matrix $A = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix}$, Find A^{-1} . Using A^{-1} solve the system of equations $2x - 3y + 5z = 11, 3x + 2y - 4z = -5$ and $x + y - 2z = -3$

28. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

OR

Find the maximum areas of an isosceles triangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with its vertex at one end

29. There are two factories located one at place P and the other at place Q. From these locations, a certain commodity is to be delivered to each of the three depots situated at A, B and C. The weekly requirements of the depots are respectively 5, 5 and 4 units of the commodity while the production capacity of the factories at P and Q are respectively 8 and 6 units. The costs of transportation per unit are given below:

From/To	Number of hours required on machines		
	A	B	C
P	160	100	150
Q	100	120	100

How many units should be transported from each factory to each depot in order that the transportation cost is minimum? What will be the minimum transportation cost?