

SAMPLE PAPER-2015
CLASS-XII
Subject:-Mathematics

TIME: 3h

M.M : 100

GENERAL INSTRUCTION:

- (a) All questions are compulsory.
(b) This question paper consists of 29 questions divided into three section A, B, and C. Section A comprises of 10 question of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 7 questions of six marks each.
(c) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
(d) There is no overall choice. However, internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
(e) Use of calculators is not permitted. You may ask for logarithmic tables, if required.

SECTION--A

- Q1: Prove that : $\frac{9\pi}{8} - \frac{9}{4} \sin^{-1} \frac{1}{3} = \frac{9}{4} \sin^{-1} \frac{2\sqrt{2}}{3}$
Q2: Give example of two functions $f: N \rightarrow Z$ and $g: Z \rightarrow Z$ such that gof is injective but g is not injective.
Q3: If a matrix has 18 elements, what are possible orders it can have? What, if it has 5 elements?
Q4: If A is an invertible matrix of order 2, then find $\det (A^{-1})$.
Q5: Find $|\vec{a}|$ and $|\vec{b}|$ if $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 8$ and $|\vec{a}| = 8|\vec{b}|$
Q6: Show that $f(x) = e^{2x}$ is strictly increasing on R.
Q7: Find, $|\vec{a} \times \vec{b}|$, if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$.
Q8: Show that the line through the points (4, 7, 8), (2, 3, 4) is parallel to the line through the points (-1, -2, 1), (1, 2, 5).
Q9: Write the order and degree of the differential equation, $y = x \frac{dy}{dx} + a \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$
Q10: State Lagrange's mean value theorem.

SECTION-B

- Q11: Show that the relation R defined by $R = \{(a, b) : (a - b) \text{ is divisible by } 3; a, b \in N\}$ is an equivalence relation.
Q12: Prove that $\cos(\tan^{-1}(\sin(\cos^{-1} x))) = \sqrt{\frac{x^2+1}{x^2+2}}$.
OR
Prove that : $\tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{7} + \tan^{-1} \frac{1}{8} = \frac{\pi}{4}$

- Q13: Using properties of determinants, prove that

$$\begin{vmatrix} x & x^2 & 1 + px^3 \\ y & y^2 & 1 + py^3 \\ z & z^2 & 1 + pz^3 \end{vmatrix} = (1 + pxyz)(x - y)(y - z)(z - x), \text{ where } p \text{ is any scalar.}$$

Q14: Differentiate $x^x + x^a + a^x + a^a$ w.r.t. x . ($a > 0, x > 0$)

Q15: If $x^2 + y^2 = t - \frac{1}{t}$ and $x^4 + y^4 = t^2 + \frac{1}{t^2}$, then prove that $x^3 y \frac{dy}{dx} = 1$.

OR

Differentiate $\tan^{-1} \left[\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}} \right]$ with respect to $\cos^{-1}(x^2)$

Q16: Find the values of a and b so that the function $f(x) = \begin{cases} x + a\sqrt{2} \sin x, & 0 \leq x < \frac{\pi}{4} \\ 2x \cot x + b, & \frac{\pi}{4} \leq x \leq \frac{\pi}{2} \\ a \cos 2x - b \sin x, & \frac{\pi}{2} < x < \pi \end{cases}$

is continuous for $0 \leq x \leq \pi$.

Q17: Evaluate: $\int \frac{e^x}{(1+e^x)(2+e^x)} dx$

Q18: Evaluate: $\int_0^1 \cot^{-1}(1-x+x^2) dx$

OR

Evaluate: $\int_0^{\frac{\pi}{2}} \sqrt{\tan x} + \sqrt{\cot x} = \sqrt{2} \pi$

Q19: Evaluate: $\int \sin^{-1}(\log x) dx$

Q20: If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, find a vector \vec{c} such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$

Q21: Find the equations of the line passing through the point $(-1, 3, -2)$ and perpendicular to each of the

line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$

OR

Find the shortest distance between the following lines: $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ and $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$

Q22: There is a group of 50 people who are patriotic out of which 20 believe in non-violence. Two persons are selected at random out of them, write the probability distribution for the selected persons who are non-violent. Also find the mean of the distribution.

Explain the importance of Non-violence in patriotism.

SECTION-C

Q23. The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method.

Q24. Using integration compute the area bounded by the lines $x + 2y = 2$, $y - x = 1$ and $2x + y = 7$.

Q25. If length of three sides of a trapezium other than base are equal to 10cm, then find the area of the trapezium when it is maximum.

OR

Show that normal at any point θ to the curve $x = a \cos \theta + a \theta \sin \theta, y = a \sin \theta - a \theta \cos \theta$ is at a constant distance from the origin.

Q26. Show that the general solution of the differential equation

$$\frac{dy}{dx} + \frac{y^2 + y + 1}{x^2 + x + 1} = 0 \text{ is given by } (x + y + 1) = A(1 - x - y - 2xy), \text{ where } A \text{ is parameter}$$

Q27. Find the distance of the point $(2, 3, 4)$ from the line $\frac{x+3}{3} = \frac{y-2}{6} = \frac{z}{2}$ measured \parallel to the plane $3x + 2y + 2z + 5 = 0$.

Q28. Ram belongs to a very small village of J&K. In his village, there are animals like cows, oxen, buffaloes, goats etc in each house. So he thinks to start a business of selling fodder machines. He has only `5760 to invest and a space for at most 20 items. An automatic machine costs him `360 and a manually operated machine `240. It is expected that he can sell an automatic machine at a profit of `22 and a manually operated machine at a profit of `18. Find the number of machines he should sell to maximize his profit. Keeping the rural background in mind justify the values to be promoted to for the selection of the manually operated machine.

Q29 Let X denotes the number of hours you study during a randomly selected school day. The probability that X can take the values x has the following form, where k is some unknown constant.

$$P(x) = \begin{cases} 0.1 & , \text{ if } x = 0 \\ kx & , \text{ if } x = 1 \text{ or } 2 \\ k(5 - x) & , \text{ if } x = 3 \text{ or } 4 \end{cases}$$

- (i) Find the value of k
- (ii) What is the probability that you study at least 2h? Exactly 2h? At most 2h?

Why early morning is considered as the best time to study, explain?

OR

There are three boxes, the first one containing 1 white, 2 red and 3 black balls, the second one containing 2 white, 3 red and 1 black and the third one containing 3 white, 1 red and 2 black balls. A box is chosen at random and from it two balls are drawn which are found to be one red and other white. What is the probability that these come from the second box?

Explain the meaning of three colours of our national flag.