General Instructions:

1. All question are compulsory.
2. The question paper consists of 29 questions divided into three sections A,B and C. Section – A comprises of 10 question of 1 mark each. Section – B comprises of 12 questions of 4 marks each and Section – C comprises of 7 questions of 6 marks each .
3. Question numbers 1 to 10 in Section – A are multiple choice questions where you are to select one correct option out of the given four.
4. There is no overall choice. However, internal choice has been provided in 4 question of four marks and 2 questions of six marks each. You have to attempt only one If the alternatives in all such questions.
5. Use of calculator is not permitted.
6. Please check that this question paper contains 5 printed pages.
7. Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

Pre-Board Examination 2015
CLASS – XII CBSE
MATHEMATICS

Time : 3 Hours **Maximum Marks: 100**

PART – A

Q.1 If $A=\begin{bmatrix}1 & 1 & 1\\ 0 & 1 & 3\\ 1 & K & 1\end{bmatrix}$,*Find k, If Cofactor of a_{13} is twice the cofactor of a_{23}*

Ans.K=-1

Q.2 Check the monotonocity i.e increasing & decreasing of
$f(x) = \cos 2x, [\pi/2, \pi]$. Ans.increasing

Q . 3 *Let* $a = 5\hat{i} - \hat{j} + 7\hat{k}, b = \hat{i} - \hat{j} + \lambda\hat{k}$ *Find* λ *such that* $\vec{a} + \vec{b}$ *and* $\vec{a} - \vec{b}$ *are perpendicular*.

Ans.$\lambda = \pm\sqrt{73}$

Q.4 Find $(\vec{i} \times \vec{j})\, \vec{k} + (\vec{k} \times \vec{j})\, \vec{i} - (\vec{i} \times \vec{k})\, \vec{j}$ Ans.=1

Q.5 Find x such that : _ $= \int_{\sqrt{2t}}^{x} \frac{dt}{\sqrt{t2-1}} = \frac{\pi}{2}$. ANS : $-\sqrt{2}$

Q.6 If $|a \cdot b| = 4$, $|a .b| = 2$, then find $|a|2|b|2$. Ans. =20

Q.7 Find the angle made by the vector i – 4j + 8k with the z – axis.

Ans.$\theta=\cos^{-1}\left(\frac{8}{9}\right)$

Q.8 Given P(A) = 1/2, P(B) = 1/3 and P(A U B) = 2/3. Are the events A and B independent?
Ans.Check $P(A \cap B) = P(A) \cdot P(B)$ yes

Q.9 If$|A|$ = 3 *find the*$|A^{-1}|$. Ans. =$\mathbf{\frac{1}{3}}$

Q.10 Find $\int_{-\pi}^{\pi} (sin^{-93}x + x^{295})dx$.Ans. =0

PART – B

Q.11 Let A = {-1, 0, 1, 2}, B = {-4, -2, 0, 2} and *f* , *g* : *A*→ *B* be functions defined by $f(x) = x^2 - x$, $x \in$ A and $g(x) = 2\,x\left|-\frac{1}{2}\right|-1, x \in$A are *f* and *g* equal. Justify your answer. Ans. $f : g =$ {(–1,2), (0,0), (1,0), (2,2)}

Q.12 Prove that the curves $y^2 = 4ax$ and x y = c^2 cut at right angles if $c^4 = 32\ a^4$ .Ans.$x = \left(\frac{c4}{4a}\right)\frac{1}{3}$, y=$c^2\left(\frac{c4}{c4}\right)\frac{1}{3}$

OR

At what points will the tangent to the curve y = 2x3 –15x2 + 36x–21 be parallel to x-axis? Also, find the equations of tangents to the curve at those points. Solution : We have, y = 2x3 – 15x2 +36x –21 .dy/dx =4x2 –30x + 36= 6(x2 – 5x + 6) = 6(x – 2)(x – 3).As tangent is parallel to x-axis, dy/dx = 0 => 6(x – 2)(x – 3) =0 => x = 2, 3.When x = 2,y = 2(8) – 15(4) + 36(2) –21=16 – 60 + 72 – 21= 7.Therefore, point is (2, 7).When x = 3, y = 2(27) – 15(9) + 36(3) – 21 = 54 – 135 + 108 –21 = 6.Therefore, point is (3, 6).Therefore, required points are (2, 7) and (3, 6). [Ans.]Equation of tangent at (2, 7) is [Using : y – y1 = m (x – x1)]y – 7 = 0 (x – 2)=> y – 7 = 0. [Ans.]Equation of tangent at (3, 6) isy – 6 = 0 (x – 3)=> y – 6 = 0. [Ans.]

Q.13 P*rovethat* $\begin{bmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{bmatrix}$=*(a+b+c)(a*^{2}*+b*^{2}*c*^{2}*)*. Whichtype of food we must eat? Ans: We should eat the foodcontaining low fats, high iron, high vitamins and high fibres.

Q.14
Evaluate: $\int_0^{x/2} \frac{x \sin x \cos x}{\sin 4\,x + \cos 4\,x}\,dx$. Ans. $\frac{\pi 2}{16}$

Q.15 Show that the function f(x)=$\begin{cases} \frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1} & if\ x \neq 0 \\ 0 & \end{cases}$If x=0isdiscontinuous at x = 0 . Ans. RHL = 1 & LHL = -1 $RHL \neq LHL$

Q.16 Form the differential equation of the family of circles having

radii 3 .Ans. $\frac{\left\{1+\left(\frac{dy}{dx}\right)_2\right\}_2}{\frac{d2y}{dx2}} \left[\left(\frac{dy}{dx}\right)\right]^2$+1}=9

OR

Solve the differential equation$\frac{dy}{dx}$ - 3y cotx = sin2x ; y = 2 when x =$\frac{\pi}{2}$. Ans. $\frac{y}{sin3\,x} = \frac{-2}{sin\ x} + 4$

Q.17 Evaluate: $\int_{-1}^{1}\{x + [x]\}dx$. Ans.= -1

Q.18 A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively 3/10, 1/5, 1/10 and 2/5. The probabilities that he will be late are 1/4, 1/3, and 1/12, if he comes by train, bus and scooter respectively, but if he comes by other means of transport, then he will not be late. When he arrives, he is late. What is the probability that he comes by train? 'Public transport should be encouraged.' Why? Ans:

P(e,1 A)= $\frac{\frac{3}{10}X\frac{1}{4}}{\frac{3}{10}X\frac{1}{4}+\frac{1}{5}+\frac{1}{3}+\frac{1}{10}X\frac{1}{12}+\frac{2}{5}X0} = \frac{3}{40}X\frac{120}{18} = \frac{1}{2}$

Publictransport should be encouraged because it results in less consumption of fuel therefore conservation plus less pollution . also less number of vehicles means less chances of traffic jam.

OR

A manufacturer has three machine operators A (skilled), B(semi-skilled) and C (non-skilled). The first operator A produces 1% defective items whereas the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of time, B in the job for 30% of the time and C is on the job for 20% of the time. A defective item is produced, what is the probability that it was produced by B? What is the value of skill in industries? Ans: the presence of skilled people in industries is important so that no compromise is made on the quality of work and efficient use of time is made which results in high productivity.

Q.19 Let $\vec{a}$ =2$\vec{i}$+$\vec{k}$,$\vec{b}$=$\vec{i}$+$\vec{j}$+$\vec{k}$ and $\vec{c}$=4$\vec{i}$-3$\vec{j}$+3$\vec{k}$be three vectors, find a vector $\vec{a}$which satisfies $r \times b = c \times b$ and$\vec{r}\ \vec{a} = 0$.Ans.$\vec{r}$=$\frac{1}{3}i$-$\frac{20}{3}j$-$\frac{2}{3}k$

Q.20 *Solve* $\sin^{-1}x+\sin^{-1}(1-x)\ \cos^{-1}x$= Ans. $x = 0,\frac{1}{2}$

Q.21 If $y^2 = 4ax,=$ then evaluate:
$\left(\frac{d^2y}{dx^2}\right)\cdot\left(\frac{d^2x}{dy^2}\right)$

Q.22 Evaluate: $\int\sqrt{\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)dx}$. Ans. ( ) $Cos^{-1}\sqrt{x}$+ $(\sqrt{x}-2)\sqrt{1}$-x OR$(-2\sqrt{1}-x)$ +$\sqrt{1}-x^2$–$\frac{1}{2}$sin^{-3}(2x-1)

OR

Evaluate: $\int\frac{e\ \tan^{-1}x}{(1+x^2)2}dx$ **.Solution.** $x = \tan\theta$ **;**

$dx = \sec^2\theta d\theta \int\frac{e\ \theta\ \sec^2\theta d\theta}{(1+tan^2\theta)2}dx$

$\int\left(\frac{1+cos2\theta}{2}dt\right)$=$\int\frac{e\theta}{2}d\theta$+$\int\frac{e\theta\ cos2\theta}{2}d\theta$ –$\frac{1}{2}$eθ+h= I_1=$\frac{1}{2}\int e^8$.cos2θdθ

$=\frac{1}{2}\left[e\theta.cos2\theta - \int -2\sin 2\theta.e\theta d\theta\right]$

$=\frac{1}{2}\left[e\theta.cos2\theta + 2sin2\theta.e8 - \int 2cos2\theta.e8d\theta\right]$

$=\frac{1}{2}\left[e\theta.cos2\theta + 2sin2\theta.e8\right] - 4\frac{1}{2}\int 2cos2\theta.d\theta\right]$

$I_1=\frac{1}{2}e\theta\cos2\theta$+Sin2 θ e θ-4I_1 I_1-$\frac{1}{10}e\theta\cos2\theta+\frac{1}{5}sin2\theta e\theta$

I=$\frac{1}{2}e\theta+\frac{1}{10}e\theta$cos2$\theta$+$\frac{1}{5}$sin2$\theta$e$\theta$+c

=$\frac{1}{10}$ $e\theta$[5+cos2θ+2sin 2 θ]+c

Putting in (i) we get

$=\frac{1}{10}e\tan^{-1}x\left[5+\frac{1-x^2}{1+x^2}+\frac{4x}{1+x^2}\right]$+c

PART – C

Q.23

Find the inverse of the matrix $\begin{bmatrix}-1 & 2 & 5\\ 2 & -3 & 1\\ -1 & 1 & 1\end{bmatrix}$ using elementary column transformation. If exist . Why is the importance of participating in cultural activities? Ans: By participating in cultural activities a person can increases his exposure into great extant. He has also a chance to meet new people and learn some new things. Ans.$A^{-1}=\frac{1}{7}\begin{bmatrix}4 & -3 & -17\\ 3 & -4 & -11\\ 1 & 1 & 1\end{bmatrix}$

Q.24 Suppose the reliability of HIV test is specified as follows. Ofpeople having HIV, 90% of the test detects the disease but 10% go undetected. Of people not having HIV, 99% of the test is judged HIV –ve but 1% are diagnosed as showing HIV + ve. From a large population of which only 0.1% has HIV, one person is selected at random, given the HIV test, and the pathologist reports as HIV +ve. What is the probability that the person actually has HIV? Ans. Required probability =$\frac{.001X.9}{.001X.9+.999X.01}=\frac{90}{1089}$

OR

A fair die is rolled. If 1 turns up, a ball is picked up at random from bag A, if 2 or 3 turns up, a ball is picked up at random from bag B, otherwise a ball is picked up from bag C. Bag A contains3 red and 2 white balls, bag B contains 3 red and 4 white balls and bag C contains 4 red and 5 white balls. The die is rolled, a bag is picked up and a ball is drawn from it. If the ball is red, what is the probability that bag B was picked up? Ans.

$=\frac{\frac{1}{3}X\frac{3}{7}}{\frac{1}{6}X\frac{3}{5}+\frac{1}{3}X\frac{3}{7}+\frac{1}{2}X\frac{4}{9}}=\frac{90}{293}$

Q.25 Define the line of shortest distance between two skew lines .Find the magnitude and the equation of the line of the shortest distance between the following lines:

$\frac{x}{2}=\frac{y}{-3}=\frac{z}{1}$ and $\frac{x-2}{3}=\frac{y-1}{-5}=\frac{z+2}{2}$ Ans.

S.D.=$\frac{1}{\sqrt{3}}eq\rightarrow\frac{3x-62}{3}=\frac{y+31}{1}=\frac{3z-31}{3}pt.A=\left(\frac{62}{3},-31,\frac{51}{3}\right)$,B=$\left(2t,\frac{-92}{3},\frac{32}{3}\right)$

Q.26 Using integration, find the area of the region Why vegetarian food is always preferred over the non-vegetarian food? Ans: vegetarian food is preferred over vegetarian food because it is easily digest able and provides quick energy. After taking vegetarian food the person does not feel tidy.$\{$(x,y):$|x-1| \le y \le \sqrt{5-x^2}\}$. Why vegetarian food is always preferred over the non-vegetarian food? Ans: vegetarian food is preferred over-vegetarian food because it is easily digest able and provides quick energy. After taking vegetarian food the person does not feel tidy.

$$\int_{-1}^{2}\sqrt{5-x^2\,dx}-\int_{-1}^{2}(1-x)dx-\int_{-1}^{2}(x-1)dx = \frac{-1}{2}+\frac{5}{2}\left(\sin^{-1}\frac{2}{\sqrt{5}}-\sin^{-1}\left(\frac{-1}{\sqrt{5}}\right)\right)=\frac{5\pi}{4}-\frac{1}{2}$$

Q.27 Kellogg is a new cereal formed of a mixture of barn and rice that contain at least 88 gram of protein and 36 milligram of iron .knowing that barn contain 80 gram of protein and 40 milligram of iron per kg and that rice contain 100 gram of protein and 30milligram of iron per kg, find the minimum cost of producing this new cereal if bran cost 5 per kg and rice cost 4 per kg. Ans .z = 5 x + 4 y.$X,y \ge 0, \frac{80x}{1000}+\frac{100y}{1000} \ge \frac{88}{1000}$ $i.e. 20x+25y \ge 22; \frac{40x}{1000}+\frac{30y}{1000} \ge \frac{36}{1000}$
$P(600,400) = 4.6kg$,$Q(0,1200) = 4.8kg$, $R(1100,0) = 5.5kg$

Q.28 Find the foot of the perpendicular from P(1, 2, 3) on the line$\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}$.Also obtain the equation of the plane containing the line and the point (1, 2, 3). Foot of the perpendicular = (3,5,9) the equation of plane = 18x-22y+5z+11= 0

OR

A variable plane which is at a constant distance p form the origin meets the coordinate axes in points A, B and C respectively. Through these points, planes are drawn parallel to the coordinates planes, show that locus of the point of intersection is

$\frac{1}{x^2}=\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{p^2}$.

Q.29 A cylinder of greatest volume is inscribed in a cone, show that
(i) R =$\frac{2}{3}h$ tan α (ii) H = $\frac{1}{3}h$(iii) Volume of the cylinder =$\frac{4}{27}\pi h^2$ $\tan^2\alpha$. (iv) r : R = 3 : 2. Where r, h, α are the radius, height and semi – vertical angle of the cone and R, H are the radius and height of the inscribed cylinder. Ans H = h – x cot a V=f(x)=πx^2(h-x cot a)→x=$\frac{2h\tan a}{3}$ &$H=\frac{h}{3}$