

Sample Paper- 2015
Subject: MATHEMATICS
CLASS XII

TIME: 3 HOURS

MAXIMUM MARKS: 100

SECTION- A

1. Find the principal value of $\sec^{-1}(-2)$

2. * is a binary operation on \mathbb{Q} such that $a*b = \text{L.C.M. of } a \text{ and } b$, evaluate $8*12$

3. If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$, find A^2 .

4. Evaluate : $\begin{vmatrix} a & b & c \\ a+2x & b+2y & c+2z \\ x & y & z \end{vmatrix}$

5. Simplify : $\cos \theta \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} + \sin \theta \begin{bmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{bmatrix}$

6. Evaluate : $\int \cos^{-1}(\sin x) dx$.

7. Write the value of $\int_0^{\pi} \left(\frac{1}{5+4\cos x} \right) dx$.

8. Find $|\vec{a} \times \vec{b}|$ if two vectors \vec{a} and \vec{b} are such that $|\vec{a}| = 2$; $|\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 4$

9. Find the direction cosines of y-axis.

10. If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$, are such that $\vec{a} + \lambda \vec{b}$ is perpendicular to \vec{c} , then find the value of λ .

SECTION-B

11. If $y = (\tan^{-1} x)^2$, show that $(x^2 + 1)^2 y_2 + 2x(x^2 + 1)y_1 = 2$.

12. Evaluate : $\int \frac{2x+3}{\sqrt{5-4x-x^2}} dx$

OR

Evaluate : $\int \frac{\cos 2x}{(\sin x + \cos x)^2} dx$.

13 Show that $f: \mathbb{N} \rightarrow \mathbb{N}$, given by $f(x) = \begin{cases} x + 1, & \text{if } x \text{ is odd} \\ x - 1, & \text{if } x \text{ is even} \end{cases}$

is both one-one and onto.

14 Differentiate $\sqrt{\frac{\sec x - 1}{\sec x + 1}}$ w.r.t.x
OR

If $x = a \left[\cos t + \log \left| \tan \frac{t}{2} \right| \right]$ and then $y = a \sin t$ find $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$.

15 Verify Rolle's Theorem for the function $f(x) = e^x \sin x$ in $[0, \pi]$

OR

The length x of a rectangle is decreasing at the rate of 3 cm/minute and the width y is increasing at the rate of 2cm/minute. When $x = 10$ cm and $y = 6$ cm, find the rates of change of (a) the perimeter and (b) the area of the rectangle.

16 Find the equation of the plane mid- parallel to the planes $3x - 4y + 12z = 26$ and $3x - 4y + 12z + 13 = 0$

17 Solve the differential equation $x^2y \, dy = (x^3 + y^3)dx$

OR

Solve : $xdy - ydx = \sqrt{x^2 + y^2} dx$

18 Find the distance of the point $(-1, -5, -10)$ from the point of intersecting of the lines $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$.

19 Find the distance of the point $(1, 0, 0)$ from the line $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$

20 Show that : $\begin{vmatrix} (b+c)^2 & ba & ca \\ ab & (c+a)^2 & cb \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$

21. Let X denote the number of hours you study on a Sunday. Also it is known that

$$P(X = x) = \begin{cases} 0.1, & \text{if } x = 0 \\ kx, & \text{if } x = 1 \text{ or } 2 \\ k(5-x), & \text{if } x = 3 \text{ or } 4 \\ 0, & \text{otherwise} \end{cases}$$

where k is a constant.

(a) Find the value of k .

(b) What is the probability that you study atleast two hours? Exactly two hours? Atmost two hours?

22. Prove that $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos 2x \log(\sin x) dx = \frac{1}{4} \log 2 - \frac{\pi}{8} + \frac{1}{4}$.

SECTION-C (6 X 7 = 42)

23. $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, find A^{-1} by elementary transformation method.

OR

Using matrices solve the following system of equations:

$$\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10; \quad \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10 \quad \text{and} \quad \frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13$$

24. Find the distance of the point (1, 2, 3) from the plane $x + 2y + 4z = 38$.

measured parallel to the line $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$

25. Find the area of the following region: $\{(x,y): x^2 \leq y \leq |x|\}$

26. An aeroplane can carry a maximum of 200 passengers. A profit of Rs 1000 is made on each executive class ticket and a profit of Rs 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximise the profit for the airline. What is the maximum profit?

27. So that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$.

OR

A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. Its semi-vertical angle is $\tan^{-1}(0.5)$. Water is poured into it at a constant rate of 5 cubic metre per hour. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 4 m.

28. Evaluate : $\int_0^{\pi/2} \frac{x}{\sin x + \cos x} dx$.

29. A letter known to have come from either TATANAGAR or CALCUTTA . On the envelop just two consecutive letters TA are visible .What is the probability that the letter has come from (i) Tatanagr (ii) Calcutta.