SAMPLE PAPER-2015
CLASS-XI
Subject:-Mathematics

TIME :3 HOURS **MAXIMUM MARK-100**

SECTION:A

(1X10)

1. Find the projection of $\vec{a}$ on $\vec{b}$ if $\vec{a}.\vec{b}$=8 and $\overrightarrow{b = 2\hat{\imath} + 6\hat{\jmath} + 3\kappa}$
2. Write a unit vector in the direction of $\vec{a} = 2\,\hat{\imath} - 6\hat{\jmath} + 3\,\kappa$
3. If matrix A=$[1 \quad 2 \quad 3]$ write AA', where A' is the transpose of matrix A.
4. Write the value of the determinant $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6x & 9x & 12x \end{vmatrix}$
5. Using principal value, evaluate $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$
6. Evaluate $\int \frac{sec^2 x}{3+tanx}\, dx$
7. Write the general equation of the plane passing through the point (-1,3,2)
8. Write a vector of magnitude 15 units in the direction of vector $\hat{\imath} - 2\hat{\jmath} + 2\,\kappa$
9. What is the degree of the differential equation ?$5x\left(\frac{dy}{dx}\right)^2 - \frac{d^2y}{dx^2} - 6y = \log x.$
10. If A=$\begin{bmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{bmatrix}, then\ for\ what\ value\ of\ \alpha\ is\ A\ an\ identity\ matrix$?

SECTION:B

(4x12)

11. A family has 2 children. Find the probability that both are boys, if it is known that
    (i) At least one of the children is a boy
    (ii) The elder child is a boy
12. Prove the following $\tan^{-1} x + \tan^{-1}\frac{2x}{1-x^2} = \tan^{-1}\frac{3x-x^3}{1-3x^2}$

    Or

    Prove the following :$\cos(\tan^{-1}\{\sin(\cot^{-1} x)\}) = \sqrt{\frac{1+x^2}{2+x^2}}$
13. Evaluate $\int \frac{cosx\ dx}{(2+sinx)(3+4sinx)}$

    Or

    Evaluate $\int x^2.cos^{-1} x\, dx$

14. Show that $f(x) = \begin{cases} \left(\frac{\sin x}{\tan x}\right) & if\ x < 0 \\ \frac{3}{2} & if\ x = 0 \\ \frac{\log(1+3x)}{e^{2x}-1} & if\ x > 0 \end{cases}$ is continuous at x=0

Or

If $y = \sin^{-1}(x^2\sqrt{1-x^2} + x\sqrt{1-x^4})$, $show\ that\ \frac{dy}{dx} = \frac{2x}{\sqrt{1-x^4}} + \frac{1}{\sqrt{1-x^2}}$

15. An equation* is defined on the set of positive rational number Q^+, given by a*b=ab/2, for all a,b ∈ Q+. Show that (i) * is a binary operation on Q+ (II)) find the identity element for * in Q+ (iii) find the inverse of a ∈ Q^+.
16. $evaluate\ \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin x + \cos x}{\sqrt{\sin 2x}}\, dx$
17. Find the points on the curve $y=x^3$ at which the slope of the tangent is equal to y- coordinate of the point.

    Or

    Find the equation of the normals to the curve $y=x^3+2x+6$ which are parallel to the line x+14y+4=0.
18. Show that the differential equation $(x-y)\frac{dy}{dx} = x + 2y$, is homogeneous and solve it.
19. Find the point on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance of 5 units from the point p(1,3,3).

    OR

    Find the distance of the point p(6,5,9) from the plane determined by the points A(3,-1,2),B(5,2,4),C(-1,-1,6).
20. Prove the following , using properties of determinants :

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$$

21. If $\vec{a} = \hat{\imath} - \hat{\jmath}, \vec{b} = 3\hat{\jmath} - \kappa$, $and\ \vec{c} = 7\hat{\imath} - \kappa$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d}$=1.
22. Using Lagrange's Mean Value Theorem, find a point on the parabola , $y = (x-3)^2$,where tangent is parallel to the chord joining (3,0),(5,4).

SECTION:C

(6X7)

23. Using matrices, solve the following system of equations :

$$x + 2y + z = 7, x + 3z = 11, 2x - 3y = 1$$

Or

Express the matrix $A = \begin{bmatrix} 2 & 1 & 4 \\ 3 & -4 & 1 \\ 4 & 5 & 7 \end{bmatrix}$ as a sum of a symmetric and a skew symmetric matrix.

24. A rectangular piece of tin of sides 45 cm and 24 cm is to be made into a box without a top by cutting a square piece from each corner and folding the flaps. What should be the side of the square to be cut off so that the volume of the box is maximum ?

25. A small firm manufactures gold rings and chains. The total number of rings and chains manufactured per day is at most 24. It takes 1 hour to make a ring and 30 minutes to make a chain. The maximum number of hours available per day is 16. If the profit on a ring is Rs.300 and that on a chain is Rs.190, find the number of rings and chains that should be manufactured per day, so as to earn the maximum profit. Make it as an LPP and solve it graphically.

26. Evaluate $\int_1^3 (3x^2 + 2x)dx$ as limit of sums.

Or

Using integration, find the area of the following region :

$\left\{(x, y): \frac{x^2}{9} + \frac{y^2}{4} \leq 1 \leq \frac{x}{3} + \frac{y}{2}\right\}$.

27. Write the vector equations of the following lines and hence determine the distance between them : $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$, $\frac{x-3}{4} = \frac{y-3}{6} = \frac{z+5}{12}$

28. A factory has three plants A, B, and C. Their daily production is 500,1000 and 2000 units. Out of these o.5%, 0.8% and 1% units respectively are found to be defective. An item is chosen at random and is found to be defective. What is the probability that it came from plant A ?

29. Evaluate : $\int_0^{\frac{\pi}{2}} \frac{x \; sinx \; cosx}{sin^4x + cos^4x} dx$