Sample Paper –2015

Class XII

Sub: Mathematics

Duration: 2.5 Hour **Max. Mark:100**

General Instructions:

1.All questions are compulsory.

2 The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 7 questions of six marks each.

3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

4. There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six mark each. You have to attempt only one of the alternatives in all such questions.

5. Use of calculators is not permitted. You may ask for logarithmic tables, if required.

Section-A

1. Find the intersection point of the lines $\frac{x-4}{3}=\frac{1+y}{5}=\frac{2+z}{2}$ and $\vec{r}=(4\hat{i}-\hat{j}-2\hat{k})+\lambda(3\hat{i}+5\hat{j}+2\hat{k})$,if they intersect.

2. Differentiate $\frac{d}{dx}(x^2+\sqrt[5]{\sin^{-1}x+\cos^{-1}x})$ w.r.t. 'x'.

3. Let the function $f:R\rightarrow R$ is defined as $f(x)=2x^2-1$ then find the value of f^{-1}.

4. Find the trace of the matrix $\begin{bmatrix}1 & 8 & 7\\ 6 & 6 & 5\\ 4 & 5 & 9\end{bmatrix}$.

5. Using principal values, evaluate the following $\cos^{-1}\cos\left(\frac{2\pi}{3}\right)+\sin^{-1}\sin\left(\frac{2\pi}{3}\right)$.

6. For what value of k the matrix $\begin{pmatrix}k & 2\\ 3 & 4\end{pmatrix}$ has no inverse.

7. Find the projection of $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ on $\vec{b}=2\hat{i}+\hat{j}+5\hat{k}$.

8. Evaluate $\int_{-1}^{1} e^{|x|}\ dx$

9. Evaluate $\int \frac{\sqrt{5+x^{10}}}{x^{16}}\ dx$.

10. Under what condition three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar?

Section-B

11. Find the value of λ for which these three vectors represented by points (-1, 4, -3) ,(3, λ, -5) , (-3, 8, -5) and (-3, 2, 1) are coplanar.

OR

The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with a unit vector along the sum ofvectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ.

12. Prove that $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & bc & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$ by using properties of determinants only.

13. Consider a binary operation

$*: R\times R \to R$ and $\circ: R\times R \to R$ defined as $a * b = |a - b|$ and $a \circ b = a$ $\forall$ a, b $\in$ R. Show that

$a * (b \circ c) = (a * b) \circ (a * c)$ $\forall$ a, b, c $\in$ R.

14. If $\cos^{-1}a+\cos^{-1}b+\cos^{-1}c=\pi$ then prove that $a^2+b^2+c^2+2abc=1$.

15. Check the continuity of the function $f(x)=\begin{cases} \dfrac{\sin 3x}{\tan 2x} & if\ x<0 \\ \dfrac{3}{2} & if\ x=0 \\ \dfrac{\log(1+3x)}{e^{2x}-1} & if\ x>0 \end{cases}$ at $x=0$. *Continuous efforts for betterment enhances the performance towards perfection. Comment on it.*

16. Solve $\dfrac{dy}{dx}+2y\tan x=\sin x$; $y\left(\dfrac{\pi}{3}\right)=0$.

17. Solve the following differential equation: $xdy - ydx = \sqrt{x^2+y^2}\,dx$.

18. Find the interval for the function f(x) = sin x .cos x; $0 \le x \le \dfrac{\pi}{2}$ is increasing or decreasing.

OR

Prove that the curves $y^2=4ax$ and $xy=c^2$ are orthogonal to each other, given that $c^4=32a^4$.

If $y=c^{a\sin^{-1}t}$ *and* $x=c^{a\cos^{-1}t}$ then prove that $x\dfrac{d^2y}{dx^2}+2\dfrac{dy}{dx}=0$.

OR

Differentiate x^{x^x} *w.r.t.* $\left(x^x\right)^x$.

19. *Evaluate* $\int (3x+5)\sqrt{x^2+3x+5}\ dx$

OR

$Evaluate: \int \sqrt[4]{\frac{1}{(x+a)^3 (x+b)^5}}\ dx.$

20. Find the shortest distance between the lines $\frac{x-4}{3} = \frac{1-y}{6} = \frac{5-z}{3}$ and $\vec{r} = (4\hat{i} - \hat{j} - 2\hat{k}) + \lambda(\hat{i} - 2\hat{j} - \hat{k})$.

21. Let E and F be two independent events. The probability that exactly one of them occurs is $\frac{11}{25}$ and the probability of none of them occurring is $\frac{2}{25}$. If P (T) denotes the probability of the occurrence of the event T, then find the value of P(E) and P(F).

Section-C

22. Let U_1 and U_2 are two urns such that U_1 contains 3 white and 2 red balls, and U_2 contains only 1 white ball. A fair coin is tossed. If head appears then one ball is drawn from at random from U_1 and put into U_2 .However, If tail appears then two balls are drawn at random from U_1 and put into U_2. Given that the drawn ball from U_2 is white, then find the probability that the head appeared on the coin.

23. Find the equation of the plane containing the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$.

OR

Find the equation of the plane passing through the intersection of the plane $4x - y + z = 10$ and $x + y - z = 4$ parallel to the line with direction ratio <2, 1, 1>. Find also the perpendicular distance of (1, 1, 1) from the plane.

24. Let the straight line $x = b$ divides the area enclosed by $y = (1-x)^2, y = 0$ and $x = 0$ into two parts R_1 and R_2 such $R_1 - R_2 = \frac{1}{4}$ sq. units. Find the value of b.

25. Evaluate: $\int_{\pi/3}^{\pi/2} \frac{\sqrt{1+\cos x}}{(1-\cos x)^{5/2}}\ dx$. Differentiate the wastage of seconds, integrate the number of hours in a day

OR

Evaluate $\int_0^2 e^{3x+1}\ dx,$ as the limit of a sum.

Differentiate the wastage of seconds; integrate the number of hours in a day

26. A line is passing through (8, 1). Prove that area of the triangle formed by the intercepts and the line is minimum, When sum of their perpendicular sides is $9+2\sqrt{2}$ units.
Mr. X, indulging himself for fake praise by leaking the question paper to his private students, few student have decided to left the tuitions from that teacher, what kind of character they are reflecting? Comment on it.

27. Using matrices solve the following system of equations:

$$x+y+z=6$$
$$x+2y+3z=14$$
$$x+4y+7z=30$$

28. There are two factories located one a place P and the other at place Q. From these locations, a certain commodity is to be delivered to each of the three depots situated at A, B and C. The weekly requirement of the depots are respectively 5, 5 and 4 units of the commodity while the production capacity of the factories at P and Q are respectively 8 and 6 units. The cost of transportation per unit is given below:

To / From	A	B	C
P	160	100	150
Q	100	120	100

How many units should be transported from each factory to each depot in order that the transportation cost is minimum? What will be the minimum transportation cost?