GENERAL INSTRUCTIONS:

1. All question are compulsory.
2. The question paper consists of 29 questions divided into three sections A,B and C. Section – A comprises of 10 question of 1 mark each. Section – B comprises of 12 questions of 4 marks each and Section – C comprises of 7 questions of 6 marks each .
3. Question numbers 1 to 10 in Section – A are multiple choice questions where you are to select one correct option out of the given four.
4. There is no overall choice. However, internal choice has been provided in 4 question of four marks and 2 questions of six marks each. You have to attempt only one If the alternatives in all such questions.
5. Use of calculator is not permitted.
6. Please check that this question paper contains 5 printed pages.
7. Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

Question Paper- 2015
CLASS-XII
CBSE MATHEMATICS

Time : 3 Hours **Maximum Marks: 100**

PART-A

Q1. LetA=$|a\mu|$Be a square matrix of order 3X3, and $C_{ij \text{ denote confactor}}$ of a_q in A. If $|A|=5,$ write the value of $a_{31}C_{31}+a_{32}C_{32}+a_{33}C_{33}$.Answer:5

Q.2 If A is square matrix of order 3 such that $|AdjA| = 225$. Find $|A^T|$.

Q.3 The line $y = x + 1$ is a tangent to the curve $y^2 4x =$ at the point a) (1,2) (b) (2,1) (c) (1,-2) (d) (-1,2) Ans : a

Q.4 Construct a 2· 2 matrix whose elements *aij* are given by

$$a_{ij}=\begin{cases}\frac{|-3i+j|}{2} & if i \neq j \\ (i+j)2 & \\ & if i = j\end{cases} \quad \text{Ans:}\begin{pmatrix}-4 & 1/2 \\ 5/2 & 16\end{pmatrix}$$

Q.5 Find the value of derivative of $\tan^{-1}(e^x)$ w.r.t. x at the point x = 0. Ans.1/2

NOTE Fill in the blanks in Questions 6 to 8.

Q.6 $\int \frac{sinx + cosx}{\sqrt{1+\sin 2x}} dx =$**…………** Ans: x+c

Q.7 If r= 2i^+4j^-k^ and b^=3i^-2j^+γk^ aare perpendicular each other, then λ = ... Ans : λ = - 2

Q.8 The projection of a = i^+3j^+k^ along is ……..b= 2i^-3j^+6k^ is……Ans :1/7

Q.9 The 2 vectors j^+k^ and 3i^-j+4k^ represents the two sides AB andAC, respectively of a Δ ABC. Find the length of the median through A.Ans : Median AD is given by

Q.10 Evaluate $\int_{-5}^{5}(\sin^{83} x + x^{123})$dx/ Ans:0

PART – B

Q.11 *Solve* $\sin^{-1} x + \sin^{-1}(1 - x) = \cos^{-1} x. Ans.\ x = 0, \frac{1}{2}$

OR

Solve the equation:$\sec^{-1}\frac{x}{a} - \sec^{-1}\frac{x}{b} = \sec^{-1} b - \sec^{-1} a$. Ans x=$\pm$ab

Q.12 Show that $\begin{bmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{bmatrix} = 0$ where a,b,c are in A.P.

Q.13 Evaluate: $\int_0^1 x(\tan^{-1} x)^2$dx.

Solution: I=$\int_1^1 x(\tan^{-1} x)^2$dx. =$\frac{\pi}{32}$ 2 –I, where $I_1=\int_0^1 \frac{x^2}{1+x^2}\tan^{-1} xax$

Integrating by parts, we have Now

I=$\frac{x^2}{32}[\tan^{-1} x)^2$=$\frac{1}{2}\int_0^1 \frac{x^2+1-1}{1+x^2}\tan^{-2} x\,dx$

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=$\frac{x^2}{32} = \int_0^2 \frac{x^2}{1+x^2}\tan^{-1} xdx$ =$\int_0^2 \tan^{-1} xdx - \int_0^2 \tan^{-1} xdx$

Q.14 If x=2cos$\theta - cos2\theta \& y = 2sin\theta - sin2\theta\ find\ \left(\frac{d^2y}{dx2}\right)$at $\theta = \frac{\pi}{2}$

Ans.$\left(\frac{d^2y}{dx2}\right)$at $\theta = \frac{\pi}{2}$ is $\frac{3}{8}sec\frac{3\pi}{4}cosec\frac{\pi}{4} = \frac{-3}{2}$

Q.15 If $x\sqrt{(1+y)} + y\sqrt{(1+x)} = 0$ then $\frac{dy}{dx} = \vdash \frac{1}{(1+x)^2}$.

Q.16 Water is dripping out at a steady rate of 1 cu cm/sec through a tiny hole at the vertex of the conical vessel, whose axis is vertical. When the slant height of water in the vessel is 4 cm, find the rate of decrease of slant height, where the semi vertical angle of the conical vessel is$\frac{\pi}{6}$

Solution: Given that $\frac{dv}{dt} = 1$cm^3/s, where v is the volume of water in the conceal vessel.

From the Fig.6.2. I=4cm, h=1 cos$\frac{\pi}{6} = \frac{\sqrt{3}}{2}i\ and\ r\sin\frac{\pi}{6} = \frac{t}{2}$

Therefore, v=$\frac{1}{3}\pi r^2 h = \frac{t^2}{3-4}\frac{\sqrt{3}}{2}t\ \frac{\sqrt{3}}{24}t^2$

$\frac{dv}{dt} - \frac{\sqrt{3\pi}}{8}t^2\frac{dv}{dt}$

Therefore, 1=$\frac{\sqrt{3x}}{8}16.\frac{dv}{dt}$

=$\frac{dv}{dt} - \frac{1}{\sqrt[2]{3\pi}}$cm/s

Therefore, the rate of decrease of slant height=$\frac{1}{\sqrt[2]{3\pi}}$cm/s

OR

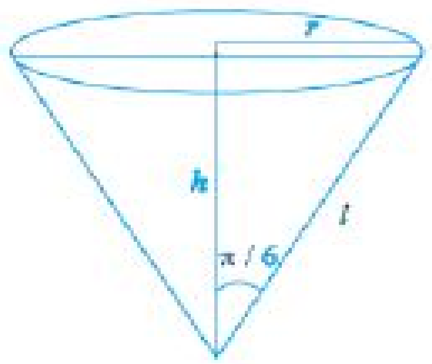

Fig. 6.2

cm/s.

Find the intervals in which the function *f* given by *f* (*x*) $= x^3 + \frac{1}{x^3}, x \neq 0$ is (i) increasing (ii) decreasing. Ans: $f(x) = x^3 + \frac{1}{x^3} = f(x)) = 3x^3 - \frac{3}{x^4} = \frac{3(x^6-1)}{x^4} - \frac{3(x^2-1)}{x^4}$

Thus f is increasing in (-∞,-1) O(1, ∞)

Thus f(x) is decreasing in (-1,0)O (0,1)

Q.17 Obtain a differential equation of the family of circles touching the x-axis at origin. Ans: Equation of circle: $x^2 + (y-a)^2 = a^2$ Required differential equation☹$x^2 - y^2$)y_1=2xy

Q.18 Evaluate: $\int \frac{ax}{(sinx + \sin 3x)}. sol$: I=$\int \frac{dx}{\sin x(1+2\,cosx)} = \int \frac{\sin x\, dx}{sin^2 x(1+2\cos x)}$

$= \int \frac{\sin x\, dx}{(1-cosx)(1+cosx)(1+2\,cosx)}$

Now differential coefficient of cosx is –sinx which is given in numerators and hence we make the substitution cos x= -sin x dx=dt

$$\therefore I = -\int \frac{dt}{(1-t)(I+t)(1+2t)}$$

We split the integrand into partial fractions

$$\therefore I = -\int \left[\frac{1}{6(1-t)2(I+t)3(1+2t)}\right] dt\ etc =$$

$$\frac{1}{6}\log(1-cosx) + \frac{1}{2}\log(1+cosx) - \frac{2}{3}\log(1+2\cos x).$$

OR

Evaluate: $\int cos2\theta log\left(\frac{cos\theta + sin\therefore}{cos\theta - sin\theta}\right) d\theta$. We know that log.$\left(\frac{cos\theta + sin\theta}{cos\theta - sin\theta}\right) = log\left(\frac{1+tan\theta}{1-tan\theta}\right) = \log tan\left(\frac{\pi}{4} + \theta\right)$

$$\int sec\theta d\theta = \log tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$$

$$\therefore \int sec\theta d\theta = \frac{1}{2} logtan\left(\frac{\pi}{4}+\theta\right)$$

$\therefore\ 2\sec 2\theta = \frac{d}{d\theta} logtan\left(\frac{\pi}{4}+\theta\right)$.......(i)

Integrating the given expression by parts, we get

I=$\frac{1}{2} sin2\theta \log tan\left(\frac{\pi}{4}+\theta\right) - \frac{1}{2}\int sin2\theta, 2sec2\theta d\theta$

=$\frac{1}{2} sin2\theta logtan\left(\frac{\pi}{4}+\theta\right) - \int tan2\theta d\theta = \frac{1}{2} sin2\theta logtan\left(\frac{\pi}{4}+\theta\right) - \frac{1}{2}\log\sec 2\theta.$

Q.19 Find the particular solution of the differential equation: (x-sin y)dy + (tany)dx = 0 : given that y = 0 when x =

The given different equation. can be written as

$\frac{dx}{dy}$+(coty)x=cosy

i.f. =e $^{coty\ dy}$=e^{log}siny=siny

∴The solution is x siny= f siny cosy dy +c

=$\frac{1}{2}\int \sin 2y\ dy + c\ or\ X\ siny = \frac{-1}{4} cos2y + c$

It is given that y=0, when x=0 ∴X siny=$\frac{1}{4}(1-cos2y) = \frac{1}{2} sin2y$

C-$\frac{1}{4} = 0 = C = 1/4$ =2x= siny is the reqd. solution.

ANS for a trip of 50 persons a bus should be used. As it has several advantage (a) it can carry many people at a time (b) it is comfortable like Volvo etc .and safe.. There can be multipleanswers to the value based questions. Students may have their ownopinion about answering them, there is no specific solution. Markswould be given for all sensible answers.

Q.20 Let *f* (*x*) = *x x* , for all *x*∈*R*. Discuss the derivability of *f* (*x*) at x = 0

Solution: We may rewrote f as f(x)-$\begin{cases} x^2, if\ x \geq 0 \\ -x^2 < 0 \end{cases}$

Now Lf^2(0)= lim$\frac{f(0+h)-f(0)}{h} - lim\frac{-h^2-0}{h} - lim - h - 0$

Now Rf^2(0)= lim$\frac{f(0+h)-f(0)}{h} - lim\frac{-h^2-0}{h} - lim - h - 0$

Since the left hand derivative and right hand derivative both are equal hence f is differentiable at x=0.

Q.21 Find the equation of the plane containing the two lines,$\underset{r}{\rightarrow}$=2i^+j^+3K^+$\gamma$(i^+2j^+5k^) and $\underset{r}{\rightarrow}$3i^+3j^+2k^+$\mu$(3i^-2j^+5k^). Find the distance of this plane from origin and also from the point

(1,1,1)ans: $\underset{n}{\rightarrow}$20i+10j-8k&$\underset{r}{\rightarrow}$(20i+10j-8k)=74→ $\underset{r}{\rightarrow}$(10i+5j-4k)=37 Hence equation of required plane is 10x+5y-4z=37 & Distance from (1,1,1) to the plane is $\frac{37}{\sqrt{141}}$ & $\frac{26}{\sqrt{141}}$.

Q.22 In answering a question on a MCQ test with 4 choices per question, a student knows the answer, guesses or copies the answer. Let ½ be the probability that he knows the answer, ¼ be the probability that he guesses and ¼ that he copies it. Assuming that a student, who copies the answer, will be correct with the probability ¾, what is the probability that the student knows the answer, given that he answered it correctly? Arjun does not know the answer to one of the questions in the test. The evaluation process has negative marking. Which value would Arjun violate if he resorts to unfair means? How would an act like the above hamper his character development in the coming years? Ans: Let A be the event that he knows the answer, B be the event that he guesses and C be the event that he copies. Then, P(A) = ½, P(B) = ¼ and P(C) = ¼ Let X be the event that he has answered correctly. Also, P(X/A) = 1, P(X/B) = ¼ and P(X/C) = ¾ Thus, Required probability =P(A/X) =

$$\frac{P(X\ A)X\ P(A)}{P(X\ A)Xp(A)+P(X\ B)XP(B)+P(X\ C)XP(C)} \quad \frac{\frac{1}{2}X1}{\frac{1}{2}X1+\frac{1}{4}X\frac{1}{4}+\frac{1}{4}X\frac{3}{4}} \frac{\frac{1}{2}}{\frac{1}{2}+\frac{1}{16}+\frac{1}{16}}=\frac{2}{3}$$

ans : If Arjun copies the answer, he will be violating the value of honesty in his character. He should not guess the answer as well as that may fetch him negative marking for a wrong guess. He should accept the question the way it is and leave it. There can be multiple answers to the value based questions. Students may have their own opinion about answering them, there is no specific solution. Marks would be given for all sensible answers.

OR

An insurance company insured 2000 cyclists, 4000 scooter drivers and 6000 motorbike drivers. The probability of an accident involving a cyclist, scooter driver and a motorbike driver are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver? Which mode of transport would you suggest to a student and why ? Ans : Let the events defined are E1: Person chosen is a cyclist
E2: Person chosen is a scoter-driver
E3: Person chosen is a motorbike driver
A: Person meets with an Accident
P (E1) = 1/6, P(E2) = 1/3, P(E3)= ½ , P (A/E1) = 0.01, P(A/E2) = 0.03,
P(A/E3) = 0.15 2 , P (E1/A) = P(A/E1).P(E1)/P(A) = 1 / 2
Suggestion: Cycle should be promoted as it is good for i. Health ii. No pollution iii. Saves energy(no petrol). There can bemultiple answers to the value based questions. Students may havetheir own opinion about answering them, there is no specificsolution. Marks would be given for all sensible answers.

PART – C

Q.23 Two schools A and B decided to award prizes to their students for three values honesty (x), punctuality (y) and obedience (z). School A decided to award a total of Rs. 11000 for the three values to 5, 4 and 3 students respectively while school B decided to award Rs. 10700 for the three values to 4, 3 and 5 students respectively. If all the three prizes together amount to Rs. 2700, then.
i. Represent the above situation by a matrix equation and form Linear Equations using matrix multiplication.
ii. Is it possible to solve the system of equations so obtained using Matrices ?
iii. Which value you prefer to be rewarded most and why? Ans :

(i) $\begin{pmatrix} 5 & 4 & 3 \\ 4 & 3 & 5 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 11000 \\ 10700 \\ 2700 \end{pmatrix}$ 5x+4y+3t=11000

4x+3y+5z=10700

x+ y+ z=2700 $1\ ^{1/2}$

(ii) Let A=$\begin{pmatrix} 5 & 4 & 3 \\ 4 & 3 & 5 \\ 1 & 1 & 1 \end{pmatrix}$ IAI=5(-2)-4(-1)+3(1)

=-10+4+3=3$\neq 0$ A^{-1}exists, so equations have a unique solution.

iii) Any answer of three values which proper reasoning will be considered correct. i.e. In my view obedience should be rewarded most , become at school lend , first of all a student should be obedient to his/her teachers , then only he/she can learn the names in school . There can be multiple answersto the value based questions. Students may have their own opinion about answering them, there is no specific solution. Marks would be given for all sensible answers.

Q.24 On the set **R**– {– 1}, a binary operation is defined by *a* * *b* = *a* + *b* + *ab* for all *a*, *b* ∈ R–{–1}. Prove that * is commutative & associative property on R – {–1}. Find the identity element and prove that every element of R – {– 1}is invertible. Ans : Hence * is commutative on R– {–1}. Identity Element : Thus, 0 is the identity element for * defined on R – {–1}. Inverse : Hence, every element of R – {–1}is invertible and the inverse of an element *a* is $\frac{a}{a+1} \in R - \{-1\}$.

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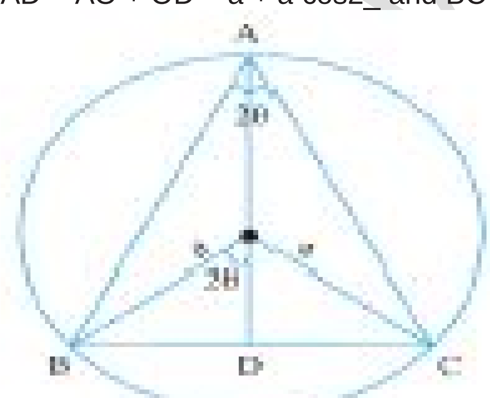
Q.25 An isosceles triangle of vertical angle 2_ is inscribed in a circle of radius *a*. Show that the area of triangle is maximum when $\theta \frac{\pi}{6}$.

Solution: Let ABC be an isosceles triangle inscribed in the circle withradius *a* suchthat AB = AC. AD = AO + OD = *a* + *a* cos2_ and BC = 2BD = 2*a* sin2 θ



Therefore, area of the triangle ABC i.e.$\Delta = \frac{1}{2} BC.AD = \frac{1}{2} 2a sin2\theta(a + acos2\theta)$=$a^2$sin2$\theta$(1+cos2$\theta$)→

$\Delta = a^2 \sin 2\theta + \frac{1}{2} a^2 \sin 4\theta$ Therefore, $\frac{d\Delta}{d\Delta} = 2a^2\cos 2\theta + 2a^2 \cos 4\theta = 2a^2(\cos 2\theta + \cos 4\,\theta)$

$\frac{d\Delta}{d\Delta}=0 \rightarrow \cos 2\theta - cos4\theta = \cos(\pi - 4\theta \frac{d\Delta}{d\Delta} = 0 \rightarrow cos2\theta = -cos4\theta = \cos(\pi - 4\theta)$

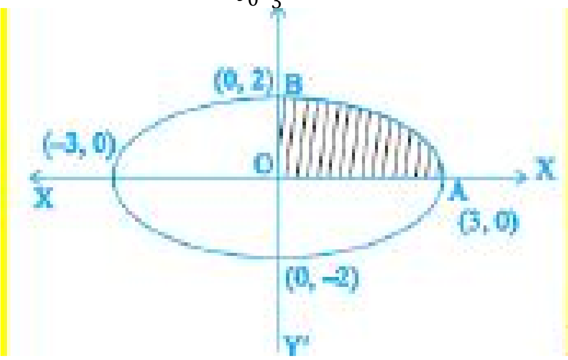
Therefore, $2\theta = \pi - 4\theta \rightarrow \theta = \frac{\pi}{6} \frac{d^2\Delta}{d\theta^2} = 2a^2(-2\sin 2\,\theta - 4sin4\theta) < 0(at\ \theta = \frac{\pi}{6})$.

Therefore, Area of triangle is maximum when$\theta = \frac{\pi}{6}$

Q.26 Draw the rough sketch of the region enclosed between the circles$x^2 +$ $y^{2=4\ and\ (x-2)^2+y^2=1}$. Using integration, find the area of the enclosed region . Ans Re**2**$\left\{\int_1^{7/4}\sqrt{1-(x-2)^2dx+^{\int_{\frac{7}{4}}^{2}\sqrt{4-x^2\ dx\}=x\ x\frac{\pi}{2}}}\right.$ $\frac{5\pi}{2}-\frac{\sqrt{15}}{2}-sin-1\left(\frac{1}{4}\right)-4sin-1\left\{\frac{7}{8}\right\}$q. unit

Find the area enclosed by the curve x=3costg, y-2 sint using integration. Solution: Eliminating t as follows: x=3 cost, y=2sint→ $\frac{x}{3}=\cos t, \frac{x^2}{9}+\frac{y^2}{4}=1$, which is the equation of an ellipse.

Required area =4$\int_0^3\frac{2}{3}\sqrt{9-x^2\ dx}=6\pi sq\ unit.$



Q.27 Find the equations of the two lines through the origin with intersect the line $\frac{x-3}{2}=\frac{y-3}{1}=\frac{z}{1}$ at angle of $\frac{\pi}{3}$. Ans.$\frac{x}{-1}=\frac{y}{2}=\frac{z}{-1}$. and$\frac{x}{-1}=\frac{y}{1}=\frac{z}{-2}$

OR

Find the projection of the line$\frac{x-`}{3}=\frac{y-3}{1}=\frac{z-4}{-5}$in the plane 2x-y+z+3=0. Ans.$\frac{x+1}{3}=\frac{y-4}{1}=\frac{z-3}{-5.}$.

Q.28 Two cards are drawn successively without replacement from well shuffled pack of 52 cards. Find the probability distribution of the number of kings. Also, calculate the mean and variance of the distribution. Why the different colors are important to us ? ANS : Let *x* denote the number of kings in a draw of two cards. Note that *x* is a random variable which can take the values 0, 1, 2. Now

P(x=0)=p(no king)= $\frac{43C2}{52C2}=\frac{\frac{48}{21(48-2)}}{\frac{52}{2(52-2)}}=\frac{48X47}{52X51}\frac{188}{221}$

P(x=1)=P(one king and one non-king)= $\frac{4C1\ 48c1}{52C2}==\frac{32}{221}$Thus the probability distribution of x is and P(x=1)=p(two kings)=$\frac{43C2}{52C2}=$ $\frac{4X3}{52X51}=\frac{1}{221}$Thus the probability distribution of x is Now mean of x=E(x)=$\sum_t^n x,p(x1)-$ $0X188221$+1X$32221+2X1221$

x	01	1	2
P x	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

(x)=E(x^2) $- [E(x)F = \left(\frac{36}{221}\right) - \left(\frac{34}{221t2}\right) = \left(\frac{6800}{221t2}\right)$ Therefore stand deviation $\sqrt{var(x)}$

$\sqrt{\frac{68000}{221}}$0.37. Ans colours are important for us become because(a) These help

Q.29 A dietician wishes to mix two types of foods in such a way that vitamin contents of the mixture c+ontains at least 8 units of Vitamin A and 10 units of Vitamin C. Food 'I' contains 2 units/kg of Vitamin A and 1 unit/kg of Vitamin C. Food 'II' contains 1 nit/kg of Vitamin A and 2units/kg of Vitamin C. It costs Rs 50 per kg to purchase Food 'I' and Rs 70 per kg to purchase Food 'II'. Formulate this problem as a linear programming problem to minimize the cost of such a mixture and solve it graphically. On the same day, a press release was also to be given innewspaper regarding the event , with the importance of sports for healthy life . you are to given some points which highlights the importance of sports for healthy life? ANS : Let the mixture contains $x$kg of food I and y kg of food II. Thus we have to minimize Z = 50x +70ySubject to : $2x + y \geq 8$; $x + 2y \geq 10$; $x, y \geq 0$.

The feasible region *determined by the above inequalities is an unbounded region. Verticesof feasible region are A (0, 8) B (2, 4*) ; *C*(*10, 0*) *.Now value of Z at A* (*0, 8*) = *50·0*+*70·8*=*560. B*(*2, 4*) = *380 ; C*(*10, 0*) = *500 . Hence, theoptimal mixing strategy for the dietician would be to mix 2 kg of food I and 4 kg of food II to get the minimum cost of the mixture i.e Rs 380*.**ANS** sports are very important to lead a healthy life. These help tokeep ourselves (a) Fresh (b) healthy (c) smarter (d) intelligent(e) fit . There can be multiple answers to the value basedquestions. Students may have their own opinion about answering them, there is no specific solution. Marks would be given for all sensible answers